

**Convegno**  
**'SCARTI MINERARI: DA RIFIUTO A RISORSA**  
**6-7 ottobre 2022**

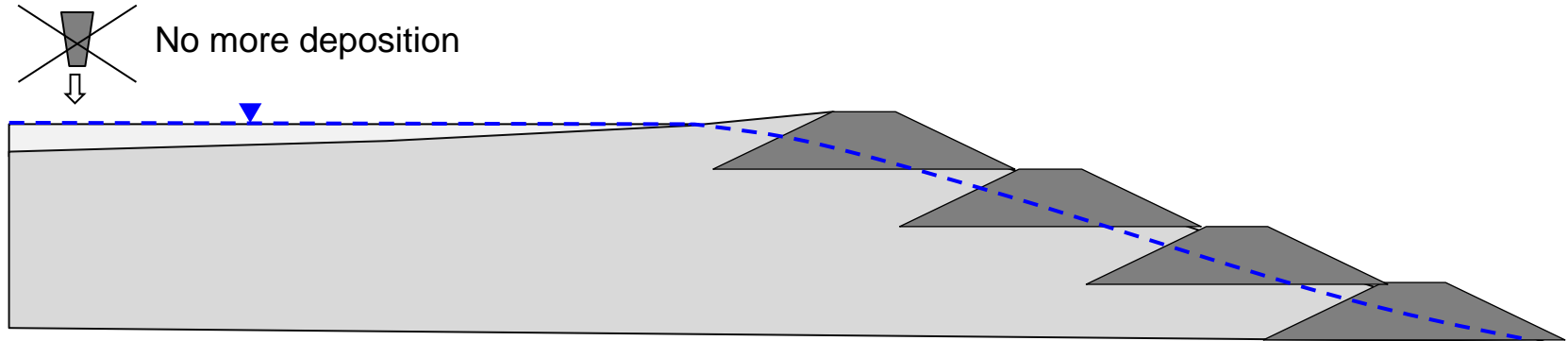
**Comportamento idraulico e meccanico dei terreni non saturi:  
Ruolo nella progettazione e gestione dei depositi di sterili minerari**

**Professor Alessandro Tarantino**



Department of Civil and Environmental Engineering  
University of Strathclyde  
Glasgow, Scotland  
[alessandro.tarantino@strath.ac.uk](mailto:alessandro.tarantino@strath.ac.uk)

# Post-closure failure hazard of Tailing Storage Facilities

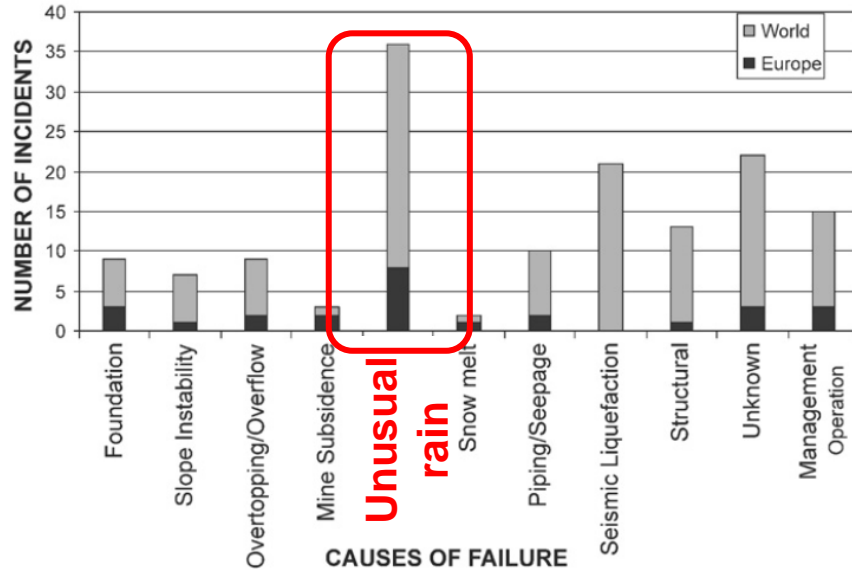


## LIQUEFACTION

- Excessive rate of loading due to dam rising  $\Rightarrow$  NO
- Excessive rate due to increase of recharge  $\Rightarrow$  NO
- Seismic loading  $\Rightarrow$  Low risk in Europe
- Transit of machinery  $\Rightarrow$  Low risk
- Failure of the drainage system  $\Rightarrow$  Medium risk
- **Other?**

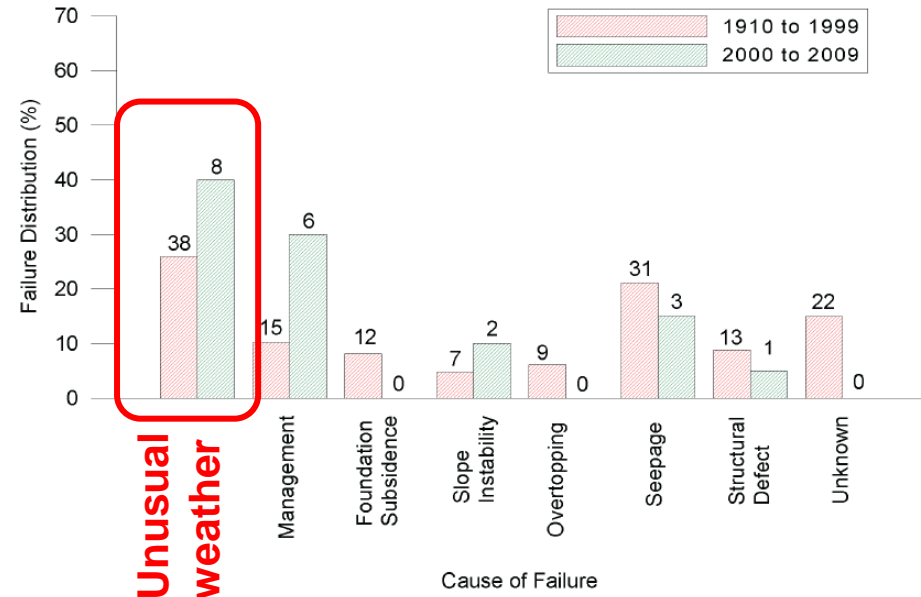
# Review of causes of failure

*Rico et al 2008*



In Europe, the most common cause of failure is related to unusual rain

*Azam and Li 2010*



Failures due to unusual rain increased from 25% pre-2000 to 40% post-2000

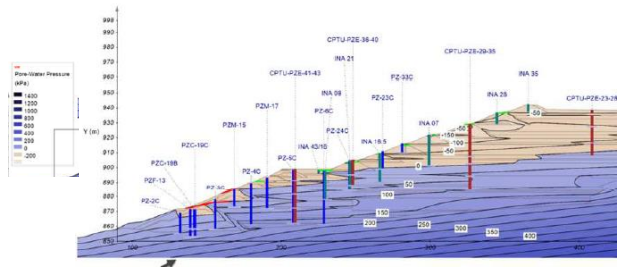
# Failure of Feijão Dam I

(Report of the Expert Panel, Robertson et al. 2019)

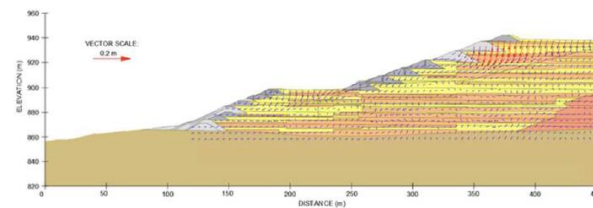
July 2016 → Cessation of tailings deposition

January 2019 → Sudden failure

*Simulated pore-water pressure*



*Simulated deformations*

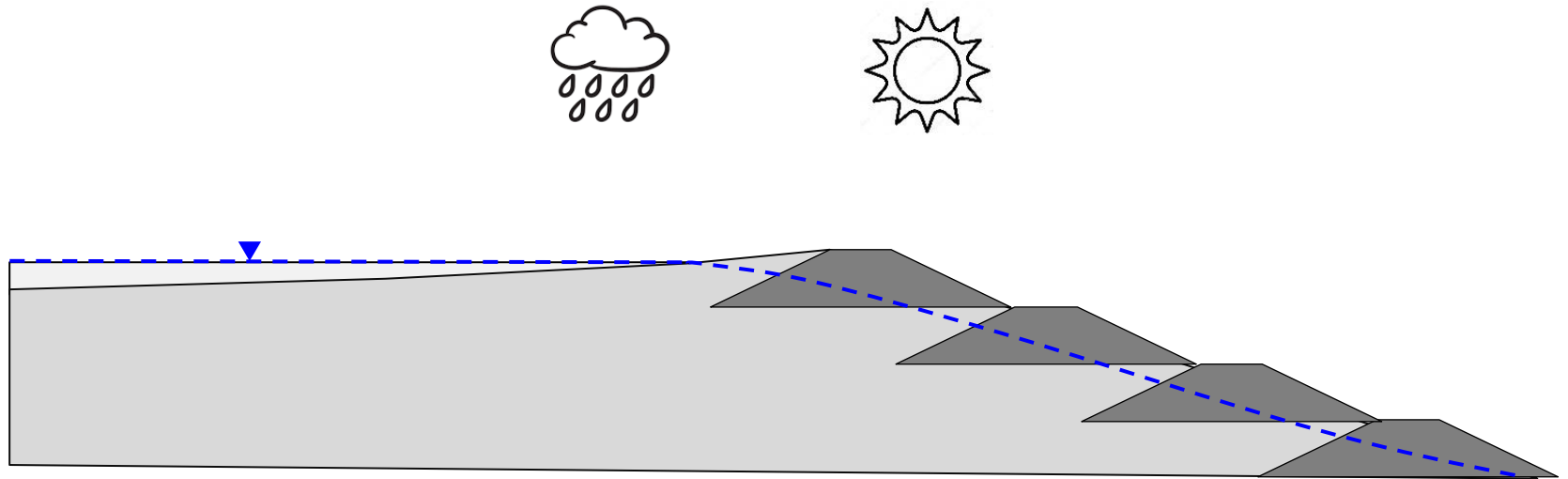


*‘Loss of suction from cumulative rainfall over the years following the cessation of tailings deposition, culminating in the intense rainfall recorded towards the end of 2018, would lead to a small strength reduction in the previously unsaturated zone (i.e., the zone above the water level)’*

*‘This analysis showed that a significant loss of suction could potentially have contributed to the observed failure mechanism’*



# Effect of climatic loading on mechanical behaviour of tailing slopes



- Why rainwater infiltration may affect slope instability
- Why suction (negative pore-water pressure) and partial saturation play a key role

# OUTLINE

## **Part 1: 'Lay' audience (no special or expert knowledge)**

- Elementary surface physical phenomena
- Capillary pressure / suction
- Sandcastles
- Rainfall-induced tailing slope instability

## **Part 2: (Geo)Technical audience**

- Water retention behaviour of unsaturated soils
- Shear strength of unsaturated soils
- Loss of shear strength due to rainwater infiltration
- Monitoring precursors variables
- Possible mitigation strategies

# **Part 1**

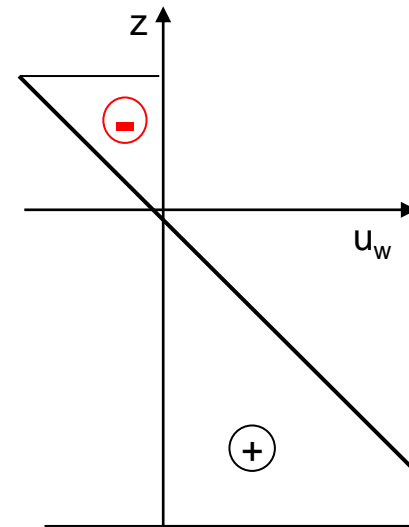
**‘Lay’ audience  
(no special or expert knowledge)**

Geomaterials above the water table are **unsaturated**  
and have **negative** pore-water pressures

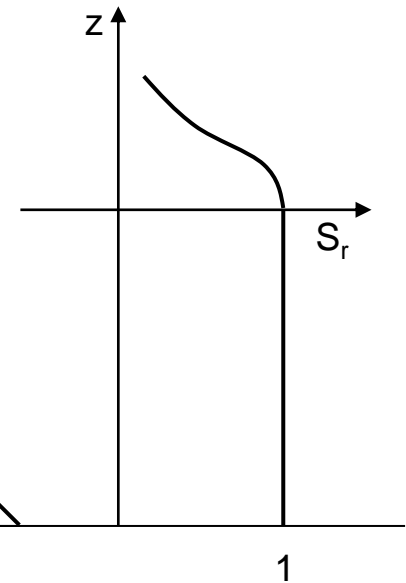
Unsaturated zone

Phreatic surface

Pore-water  
pressure

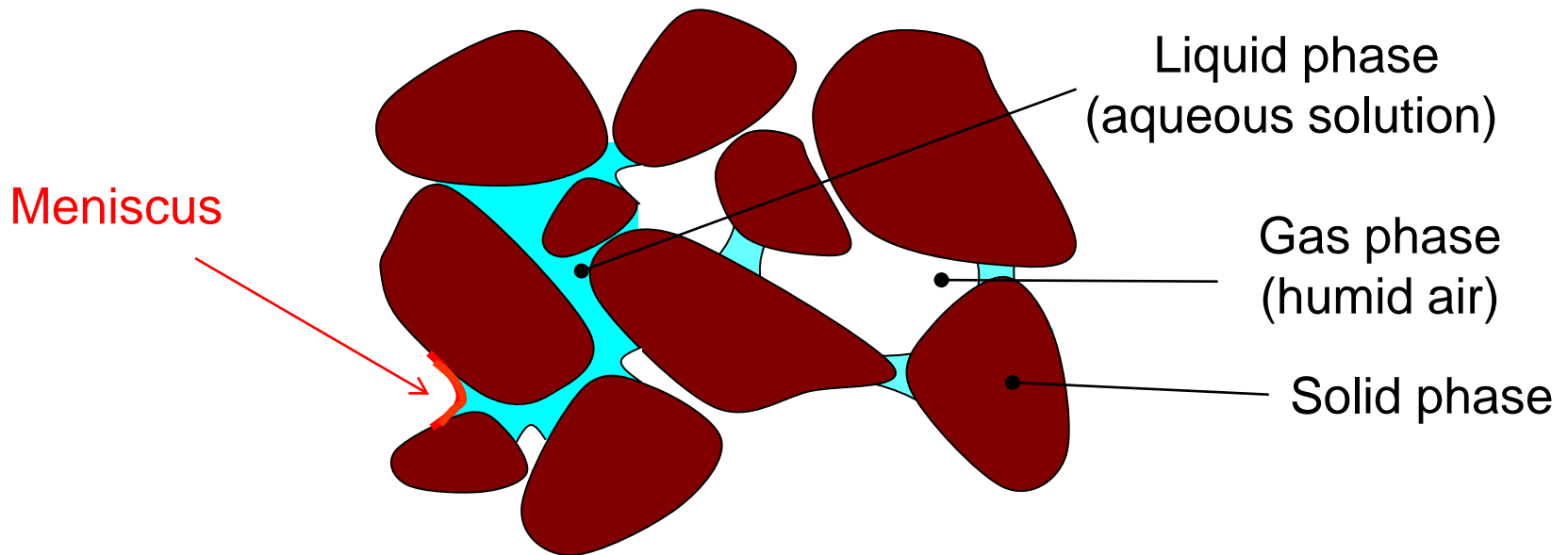


Degree of  
saturation



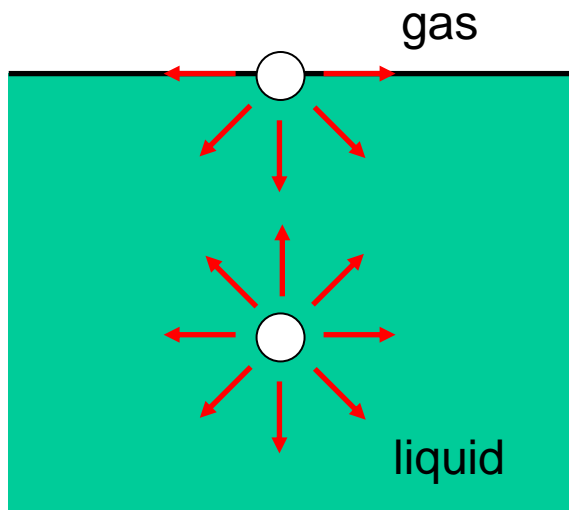
# Unsaturated state

Pore space is filled by two fluids,  
a wetting and a non-wetting one



# Cohesion and surface tension

Cohesion = Attraction force between molecules of the same type

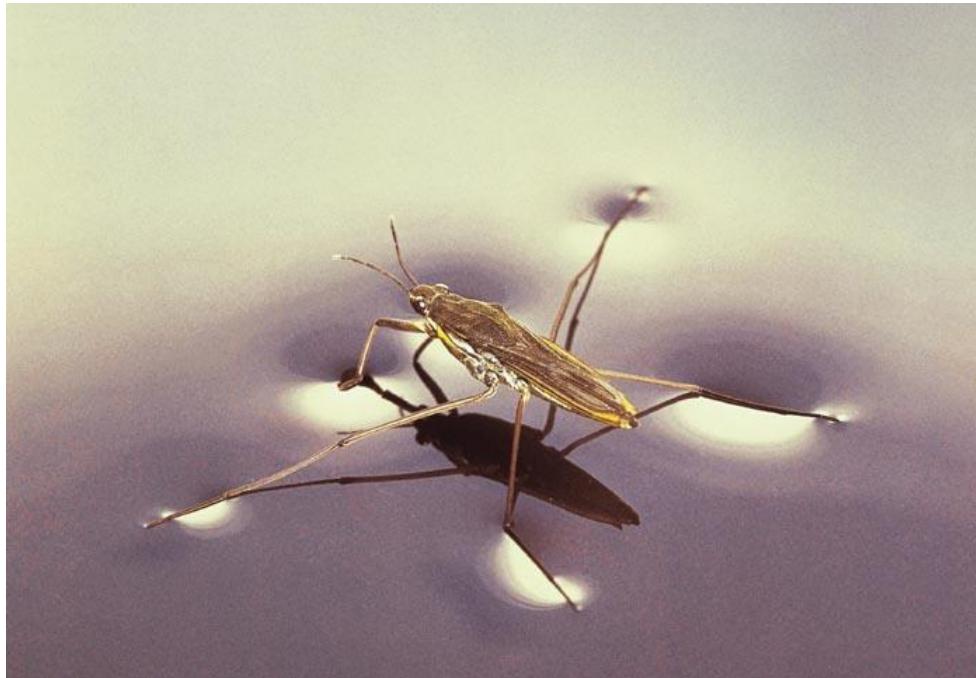


At the surface, the resultant force is directed downward

The gas-liquid interface behaves like a membrane subject to a uniform tensile stress

This stress is termed **surface tension**

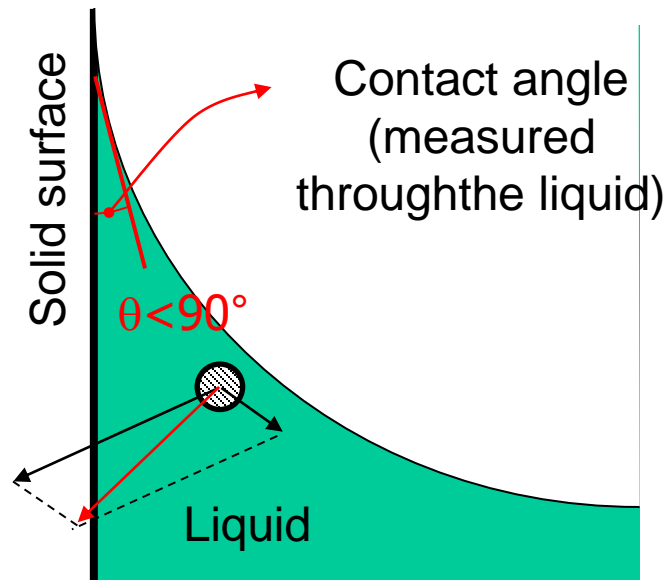
# **Real life example of surface tension: insect walking on water**



# Adhesion

Adhesion = Attraction force between molecules of different type

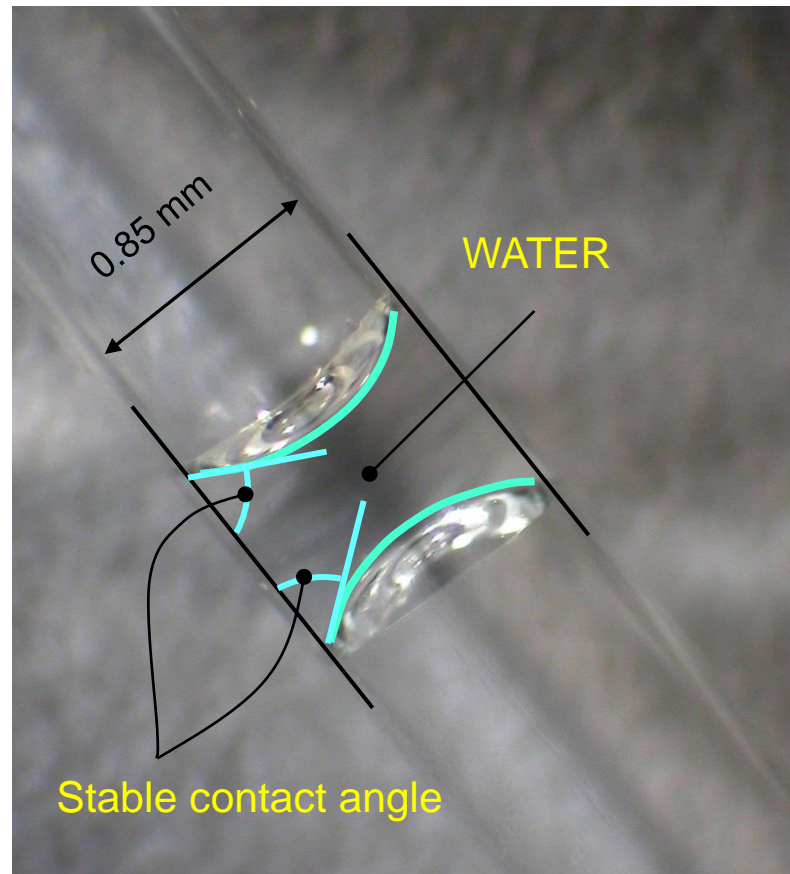
Adhesion < Cohesion



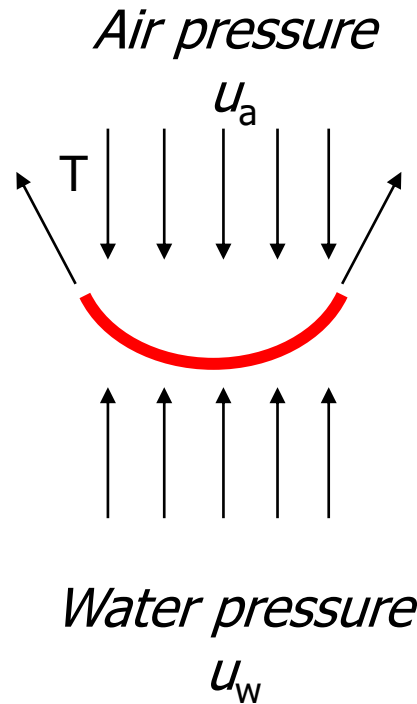
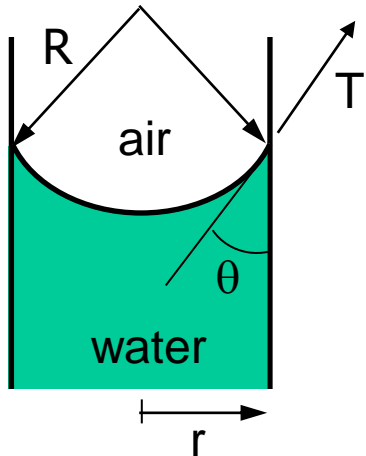
The liquid 'wets' the surface



# Real life example of adhesion: water in small diameter (capillary) tube

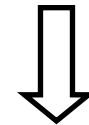


# Effect of curvature of the liquid-gas interface



**Mechanical equilibrium**

$$u_w - u_a = -\frac{2T \cos \theta}{r} = -\frac{2T}{R}$$



if  $\theta < 90^\circ$

Water pressure is **NEGATIVE**

$$u_w - u_a < 0$$

$u_a$  = air pressure [F/L<sup>2</sup>]

$u_w$  = water pressure [F/L<sup>2</sup>]

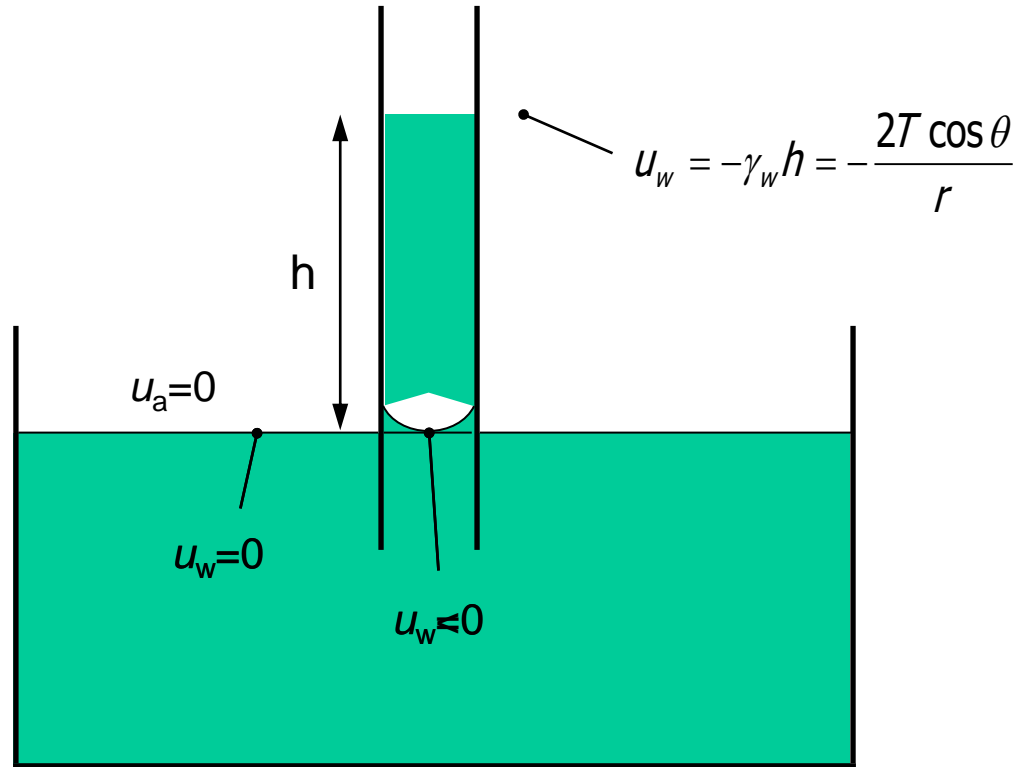
$\theta$  = contact angle

$T$  = surface tension [F/L]

$r$  = radius of capillary tube [L]

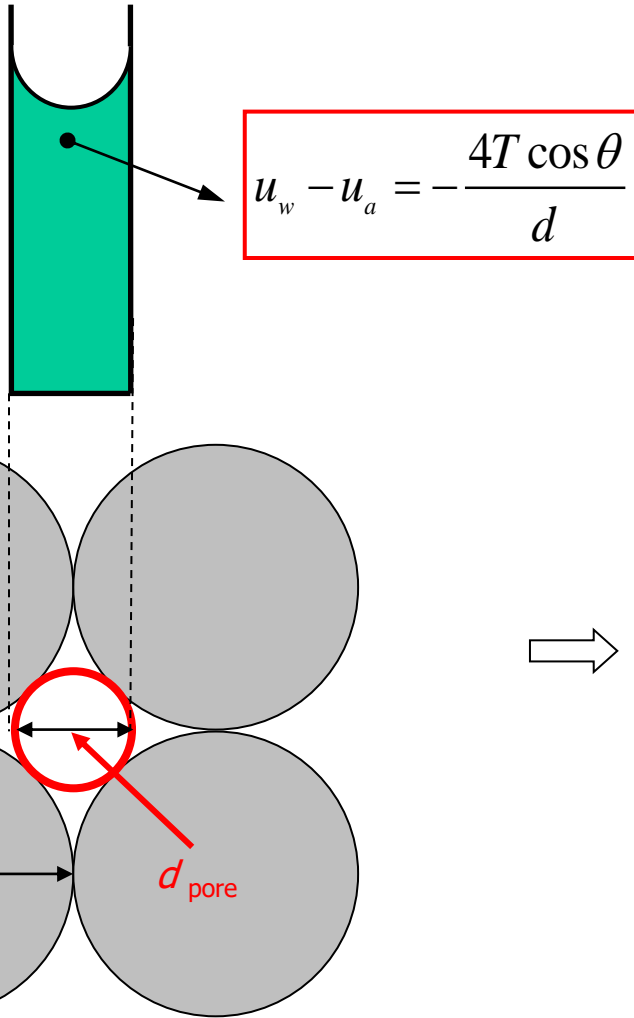
$R$  = radius of curvature of spherical cup [L]

# Real life example of surface tension: Rise in capillary tube



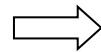
If  $\theta < 90^\circ$ , the liquid enters the cavities in the solid surface  
 $\Rightarrow$  the liquid is said to **wet** the surface

# Tensile stress of water in the capillary tube



$$d_{\text{pore}} = 1/10 d_{\text{grain}}$$

$$\begin{cases} \theta = 0 \\ T = 0.073 \text{ N/m (20°C)} \\ u_a = 100 \text{ kPa (absolute pressure)} \end{cases}$$

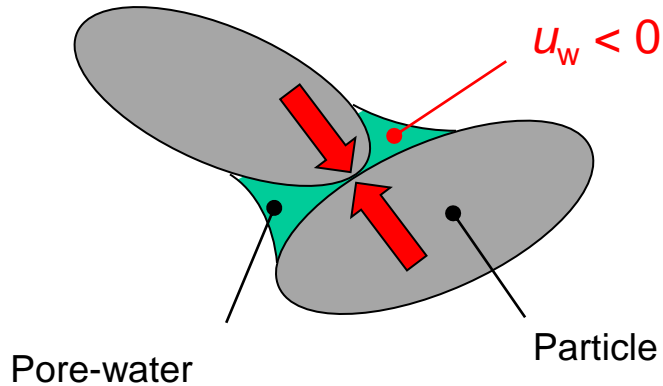


	sand ↓	silt ↓	clay ↓
$d_{\text{grain}} (\mu\text{m})$	300	30	3
$d_{\text{pore}} (\mu\text{m})$	30	3	0.3
$u_w - u_a$ (kPa)	-10	-100	-1000



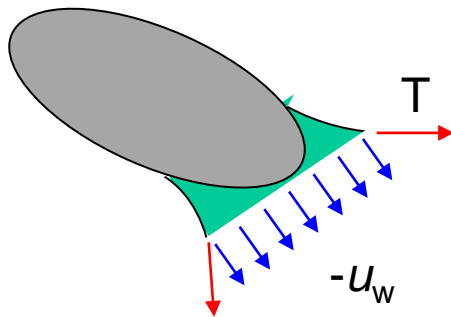
**Tailings**

# Capillary effects in soils



The contact angle of water with the particle surface is less than  $90^\circ$

The meniscus is concave toward the air side and pore water pressure is **negative**

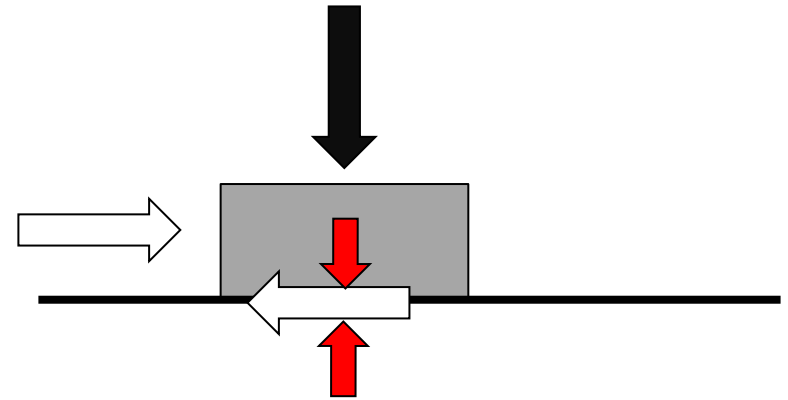


Particles are stuck together by surface tension and negative pressure

**Suction**

$$s = -u_w$$

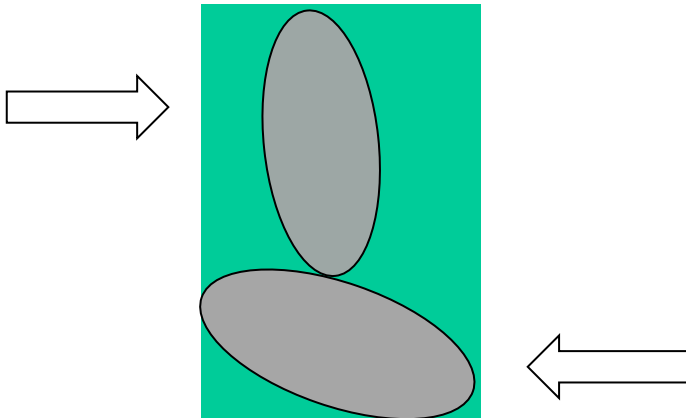
# Mechanical effects of water menisci



Saturated soil



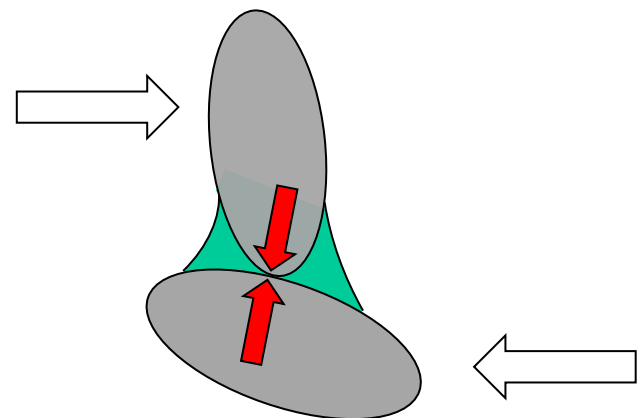
Sliding



Partially saturated soil



No sliding

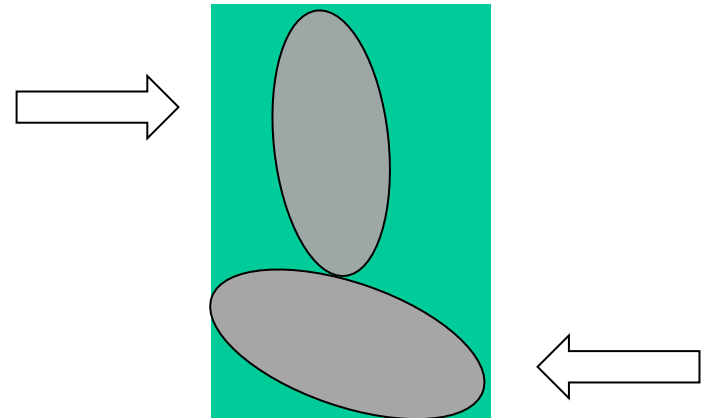
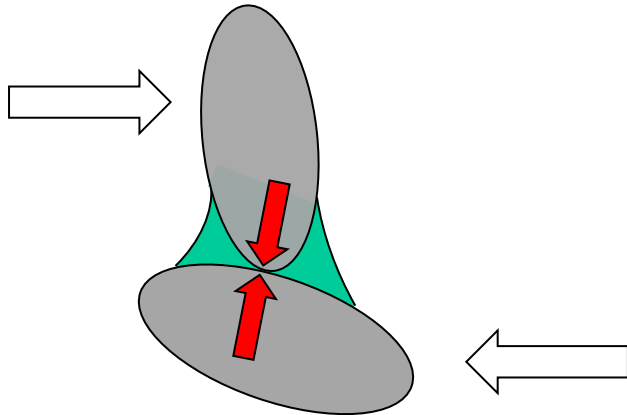
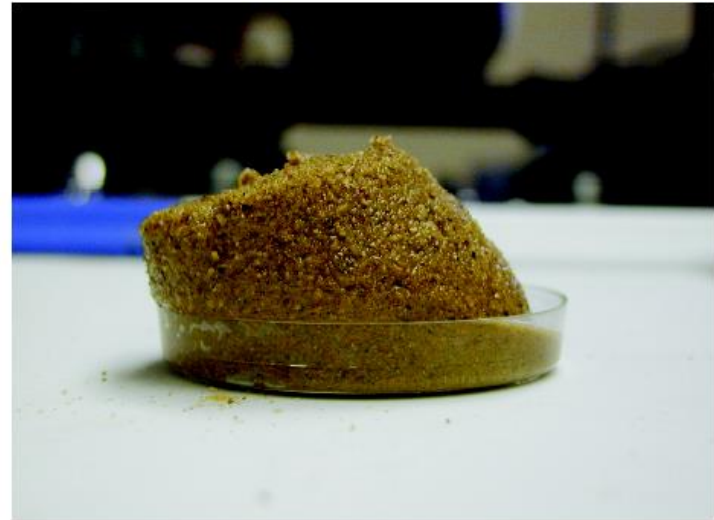


# Day-to-day experience

Medium water content  
(partially saturated)



High water content  
(quasi-saturated)



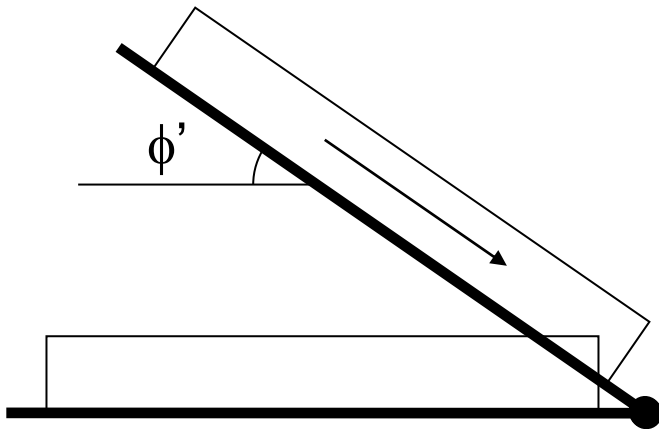
# An example of the effect of partial saturation

Dry sand / Saturated sand

No negative pressure  
No surface tension



No meniscus bonding





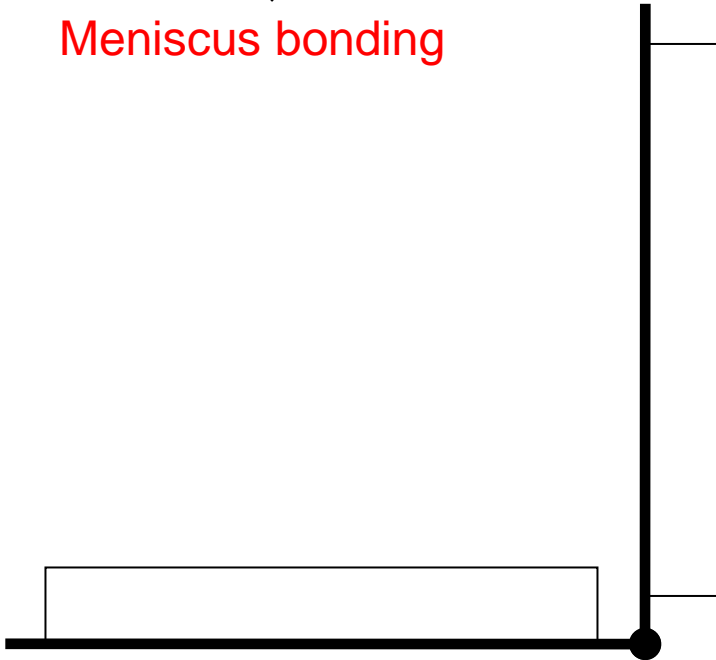
# An example of the effect of partial saturation

Partially saturated sand

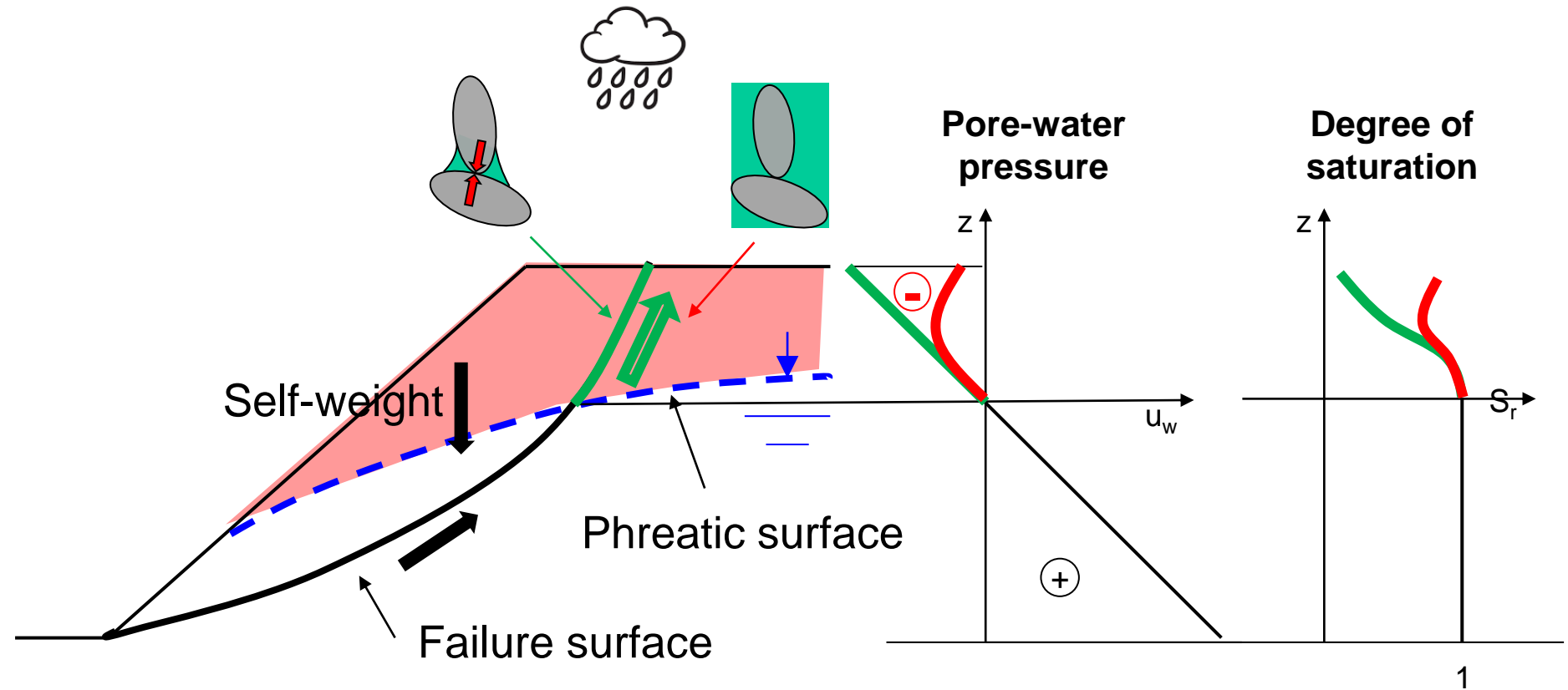
Negative pressure  
Surface tension



Meniscus bonding



# Effect of rainwater infiltration on stability of slopes



**One of the hypotheses of Feijão Dam failure**

# Part 2

## (Geo)Technical audience

*Tarantino A, Di Donna A (2019). Mechanics of unsaturated soils: simple approaches for routine engineering practice. Rivista Italiana di Geotecnica N. 4/2019*

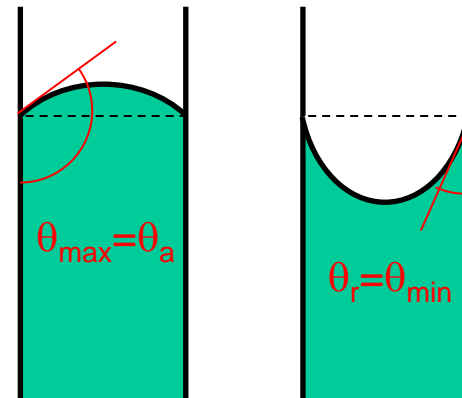
# **Water retention behaviour**

# Hysteresis of the contact angle

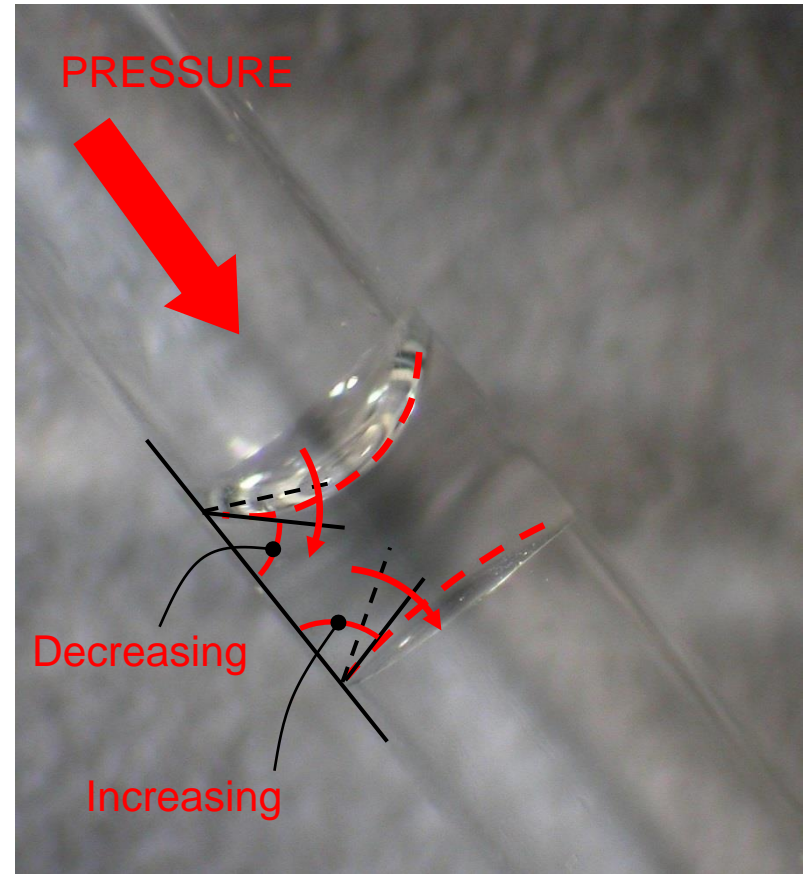
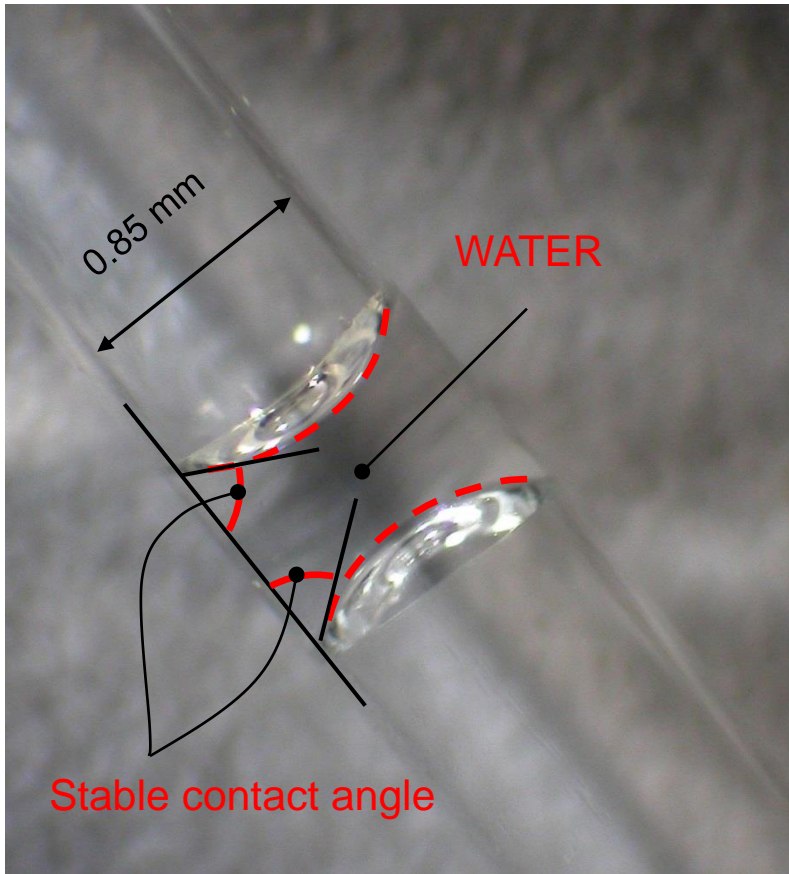
In a capillary tube, contact angle ranges from a minimum ( $\theta_r$ ) to a maximum ( $\theta_a$ )

$\theta_r$  = receding angle

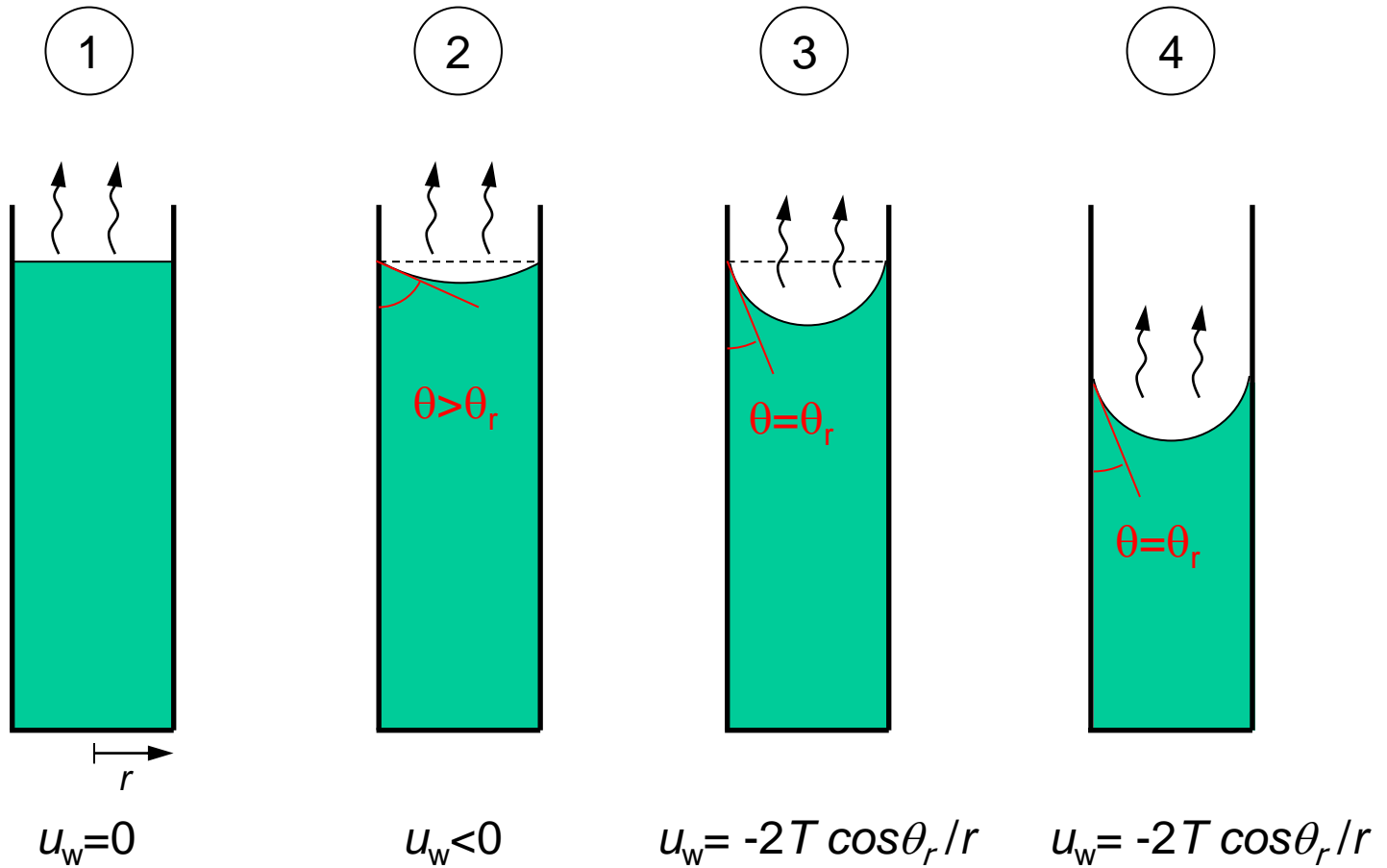
$\theta_a$  = advancing angle



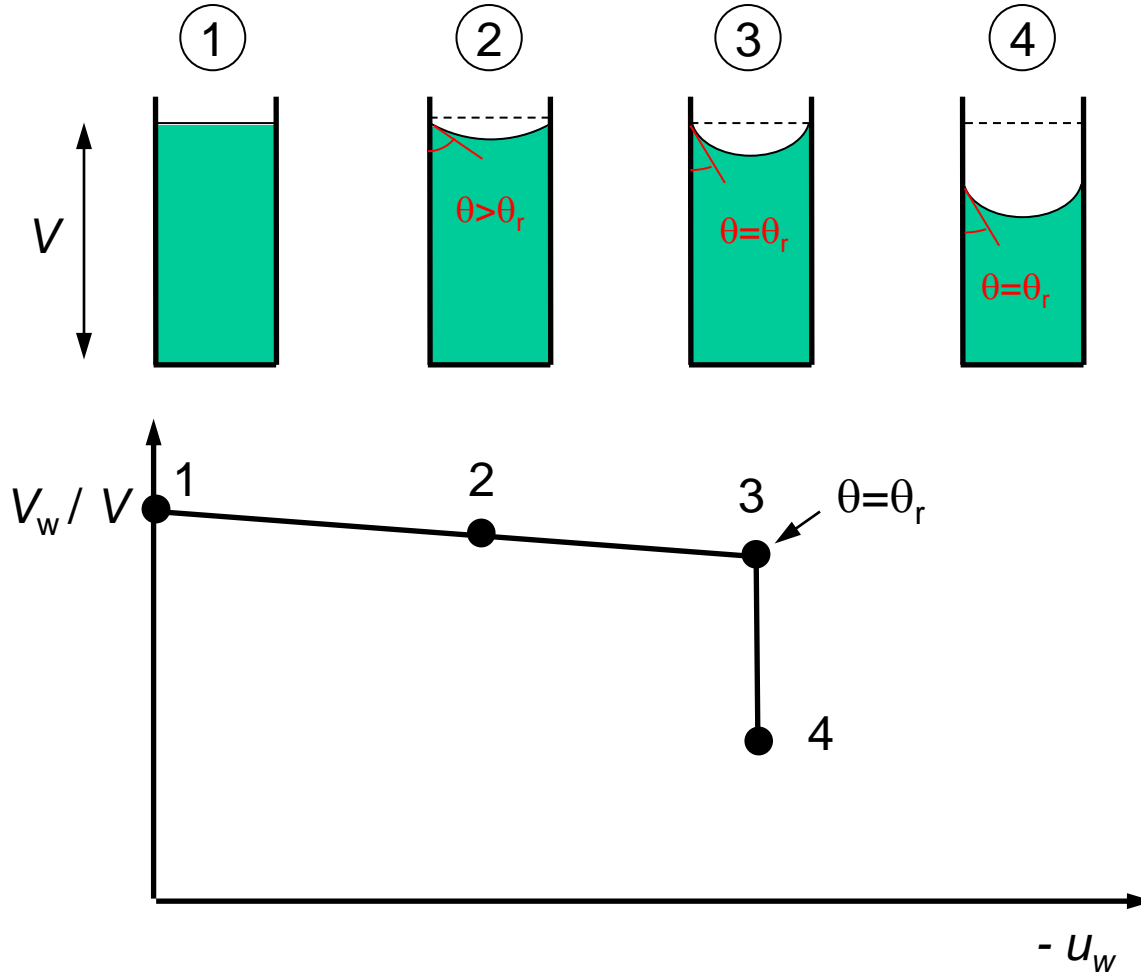
# Hysteresis of the contact angle



# Evaporation from a capillary tube

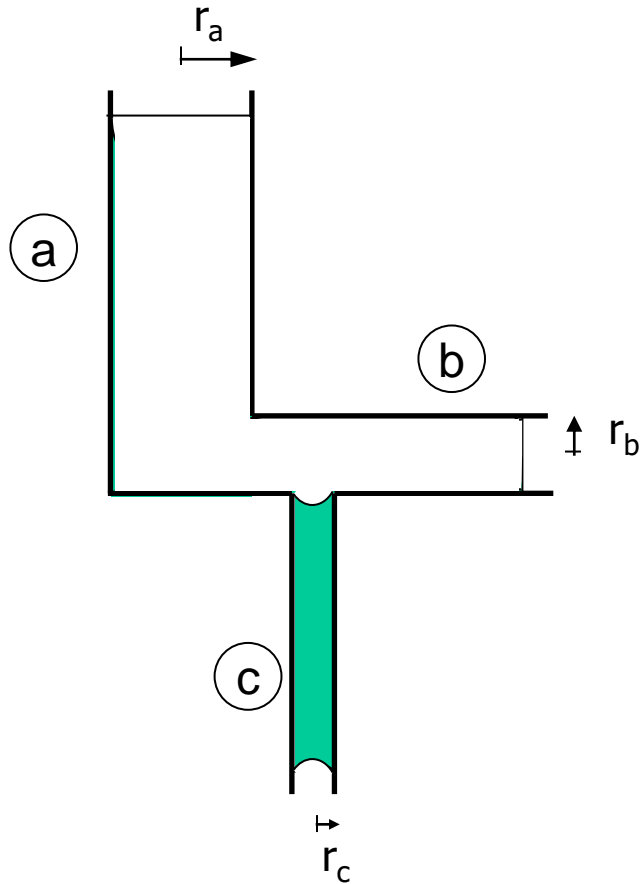


# Water retention of a capillary tube





# Evaporation from a system of capillary tubes



Meechanical equilibrium

$$U_{wa} = U_{wb} = U_{wc}$$



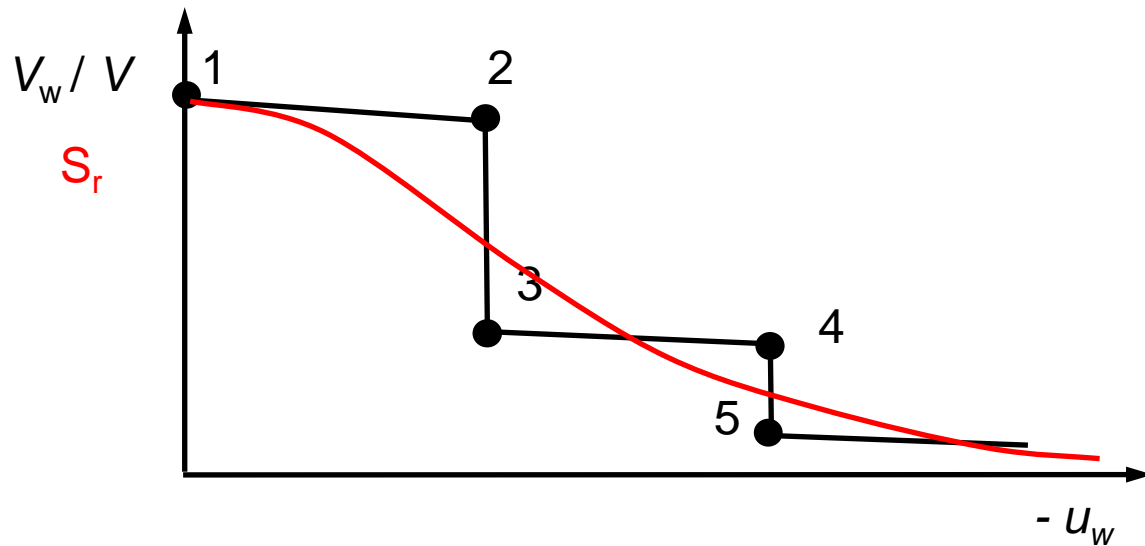
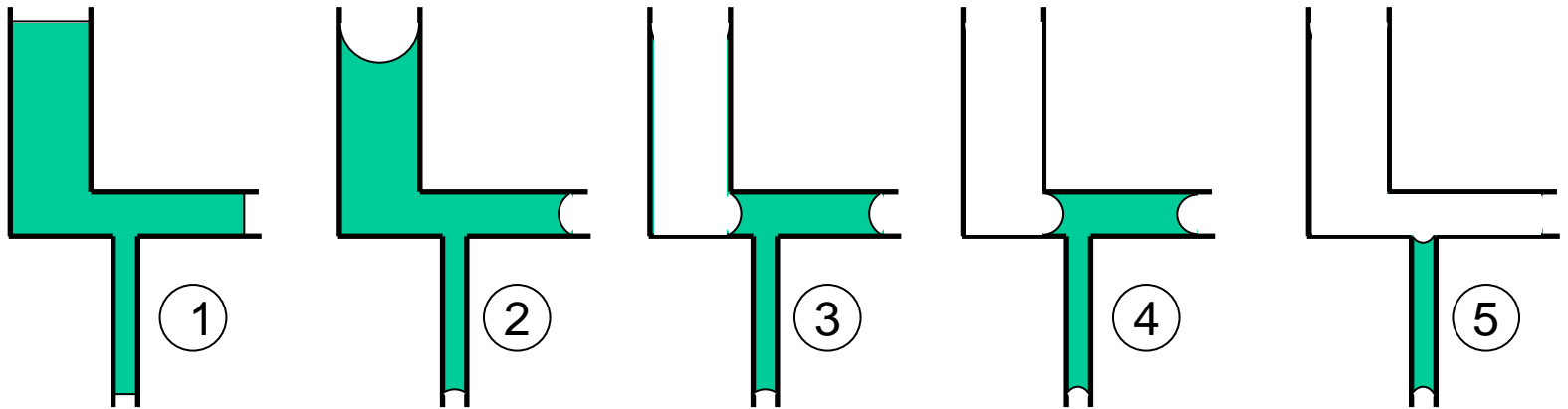
$$\frac{\cos \theta_a}{r_a} = \frac{\cos \theta_b}{r_b} = \frac{\cos \theta_c}{r_c}$$

Geometry

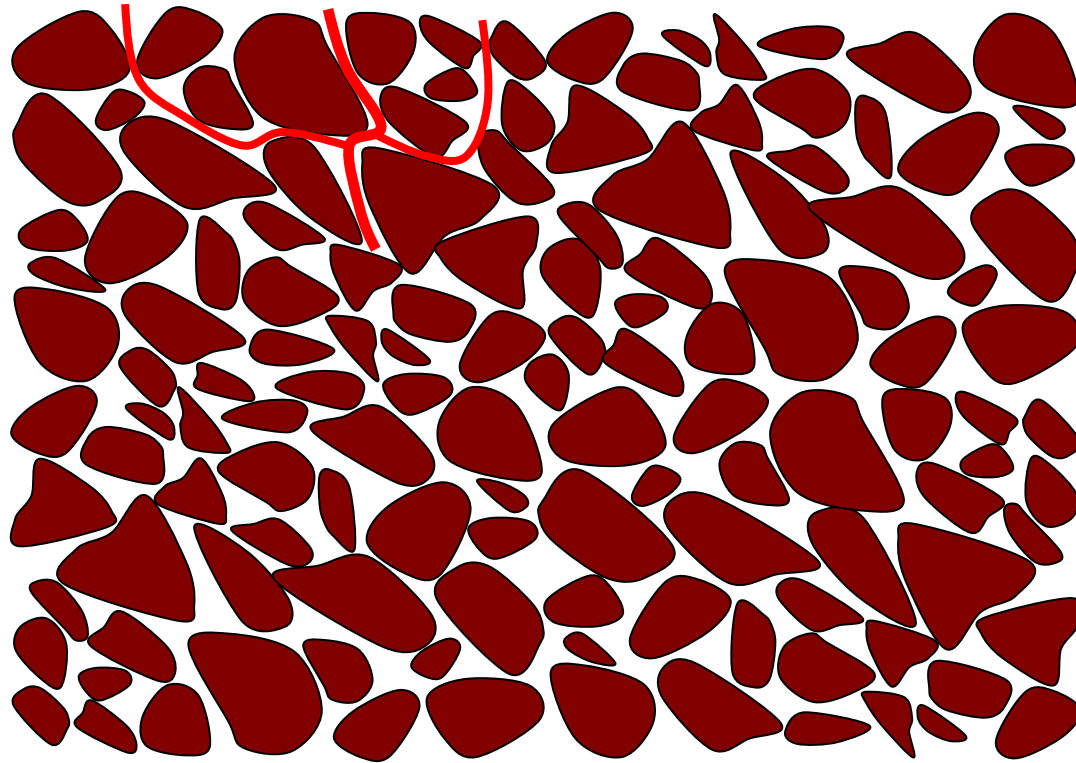
$$L_a = L_b = L_c$$

$$r_a = 2 r_b = 4 r_c$$

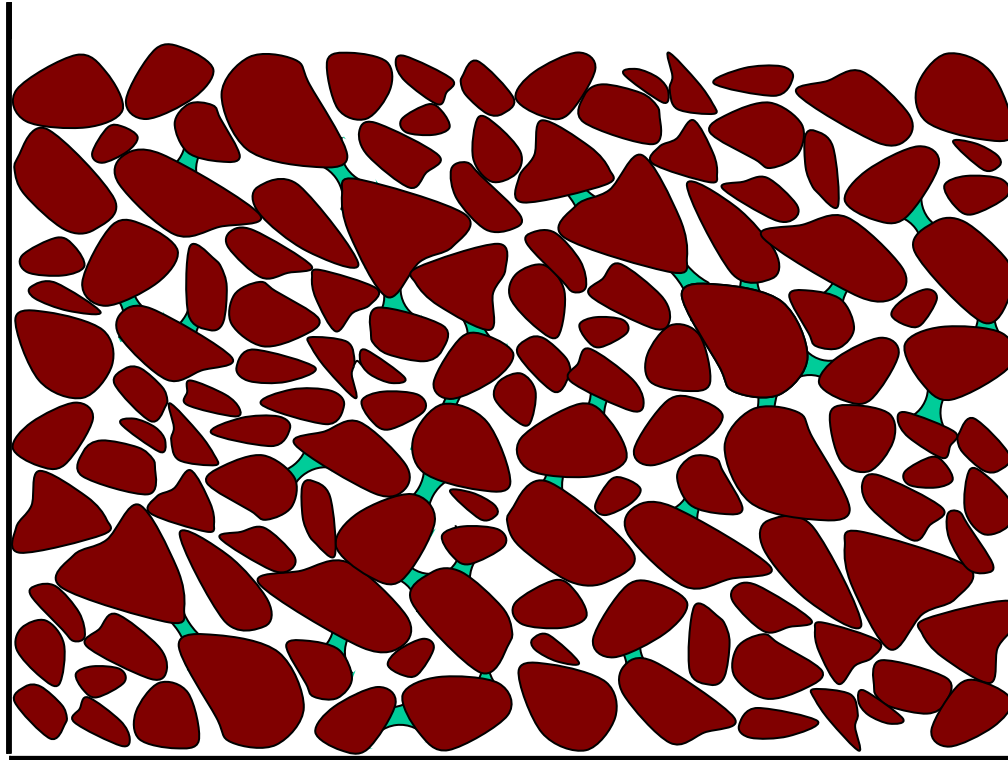
# Water retention of a sytem of capillary tubes



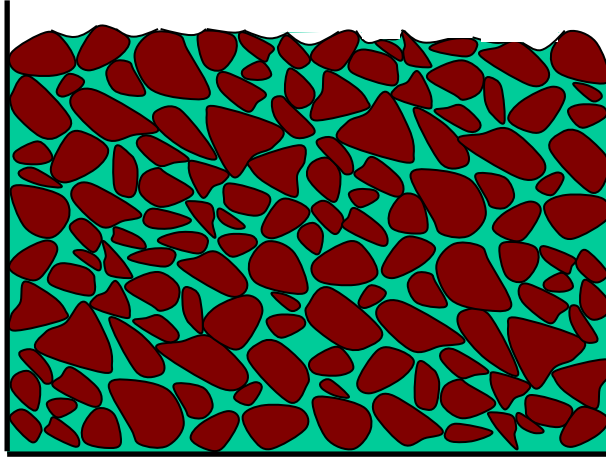
# Soil as a system of capillary tubes



# Evaporation from an initially saturated soil



# Saturated state



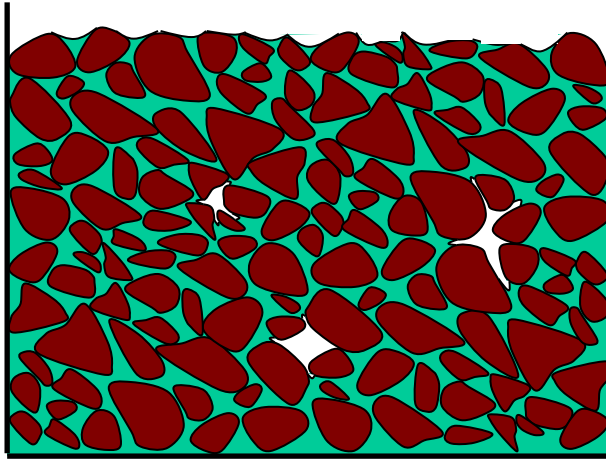
$$S = 1$$

$$u_w < 0$$

Suction is generated by the curving of menisci at the boundary

Soil is saturated, air is dissolved in water

# Quasi-saturated state



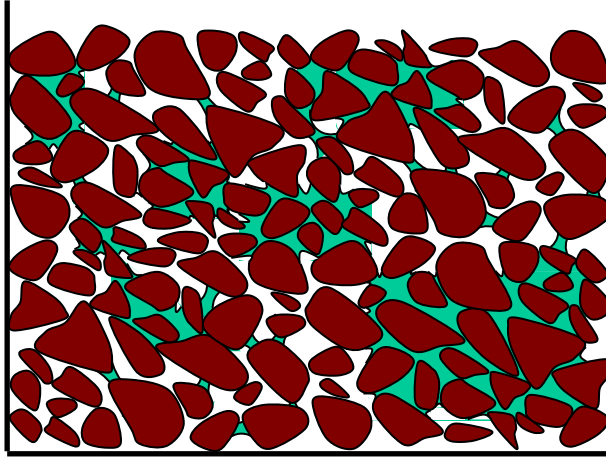
$$0.85-0.90 < S < 1$$

$$u_w < 0$$

Suction is generated by the curving of menisci at the boundary and cavities form in the pore water

Gas phase is discontinuous, liquid phase is continuous

# Partially saturated state



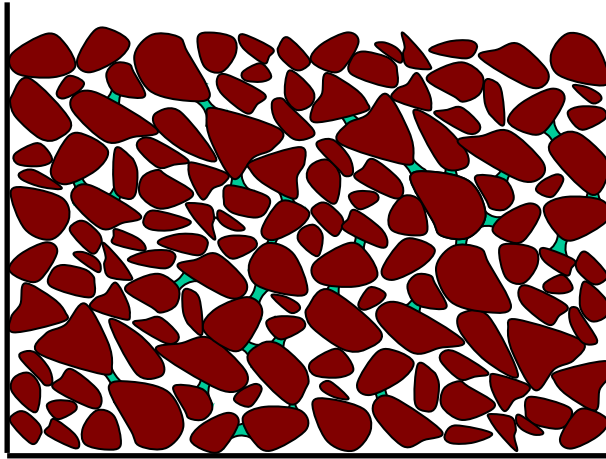
$$0.1 < S < 0.85-0.90$$

$$u_w < 0$$

Suction is generated by the curving of menisci in the pores, are saturated parts (bulk water) and part where menisci form at the interparticle contact

Gas phase is continuous, liquid phase is continuous

# Residual state



$$S < 0-0.1$$

$$u_w < 0$$

Suction is generated by the curving of menisci in the pores, and menisci form at the interparticle contact

Gas phase is continuous, liquid phase is discontinuous



# Water retention behaviour

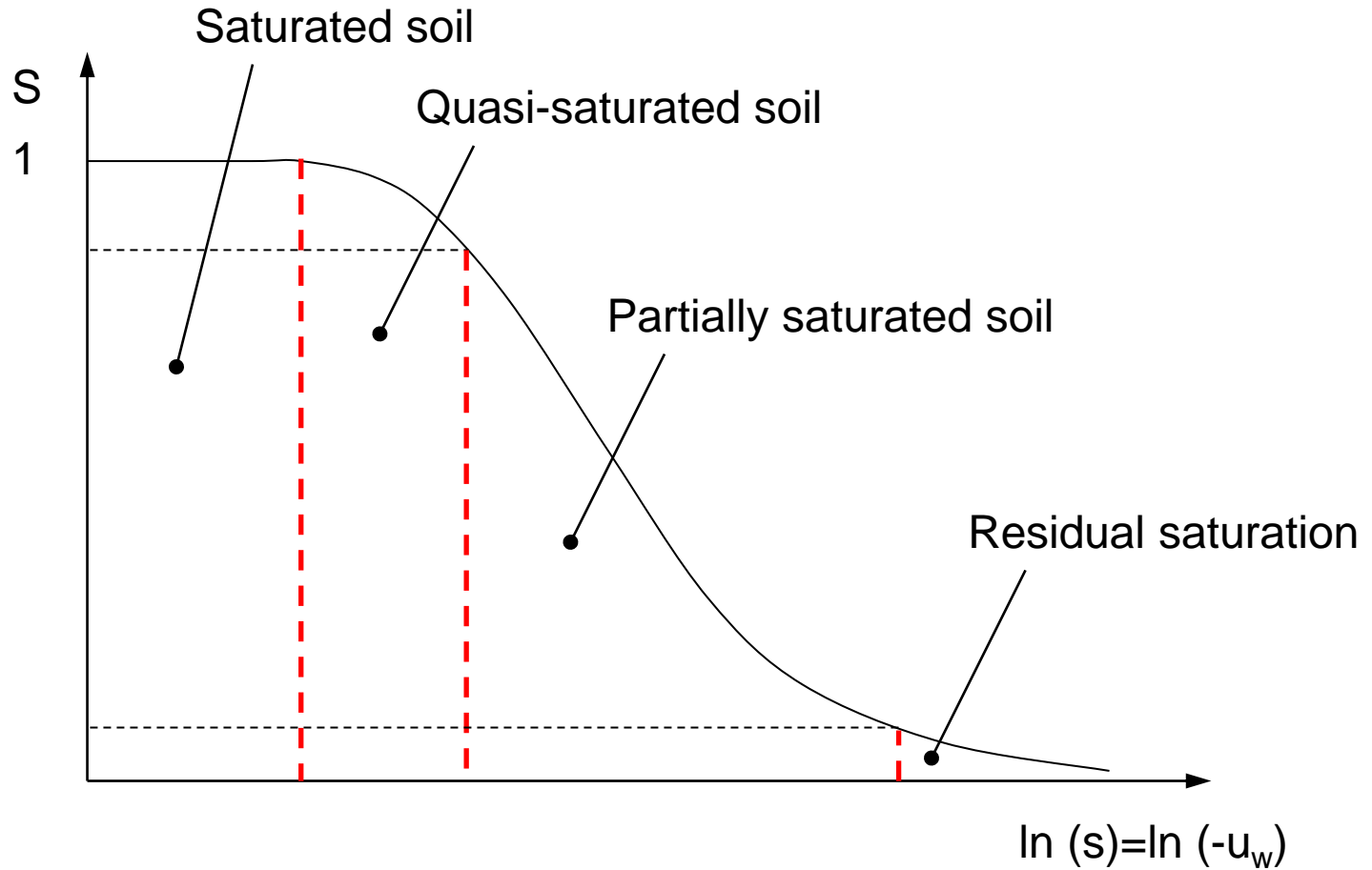
Relationship between the degree of saturation (or water content) and suction

This relationship illustrates the different state of water in the soil

It is determined in the laboratory by subjecting soil specimens to drying and wetting cycles

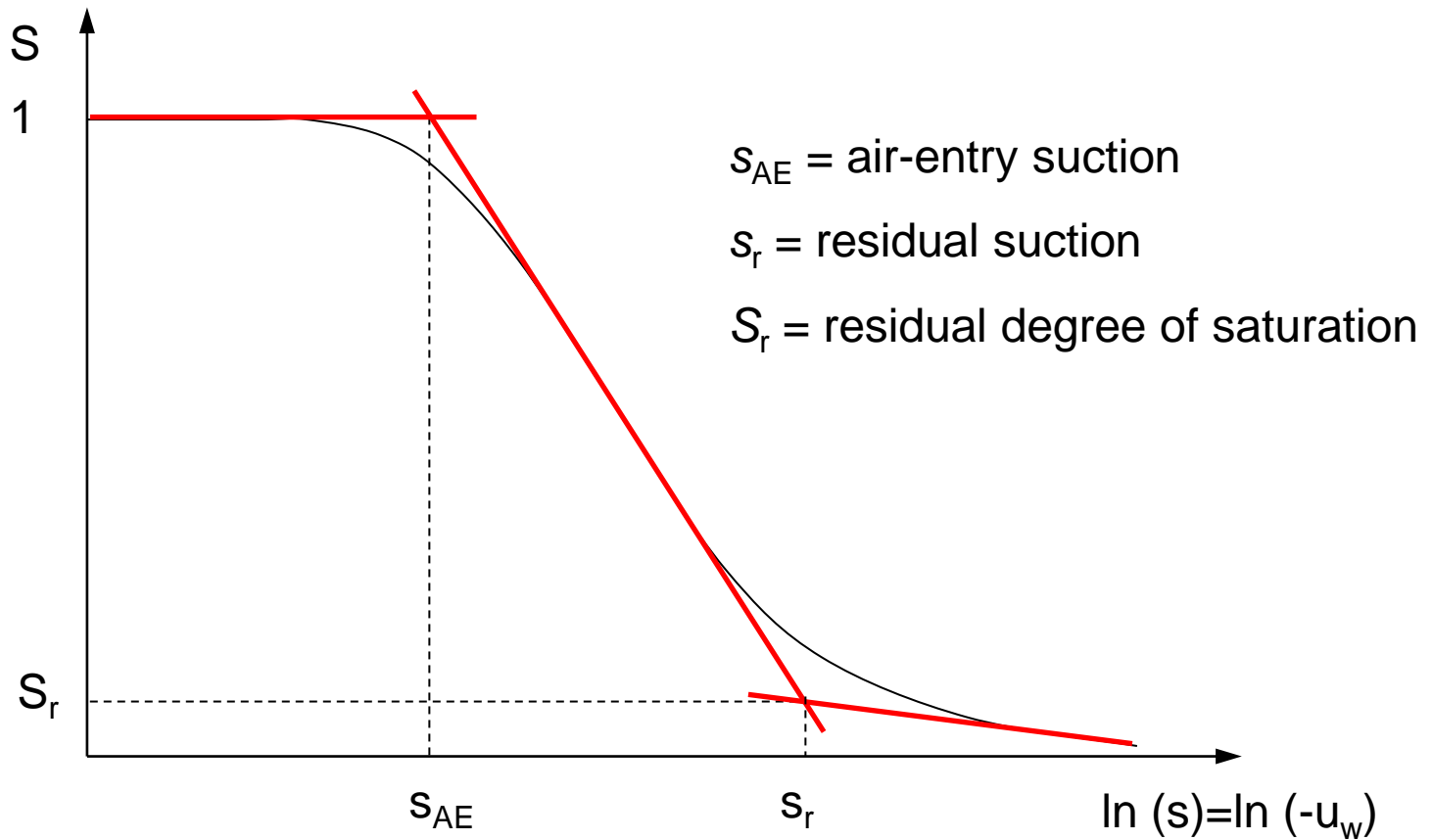
It is rarely determined in the field

# Soil water retention



Suction is the opposite of pore-water pressure,  $s = -u_w$

# Retention curve parameters



# Air-entry suction, $s_{AE}$

It is the suction where the soil desaturates

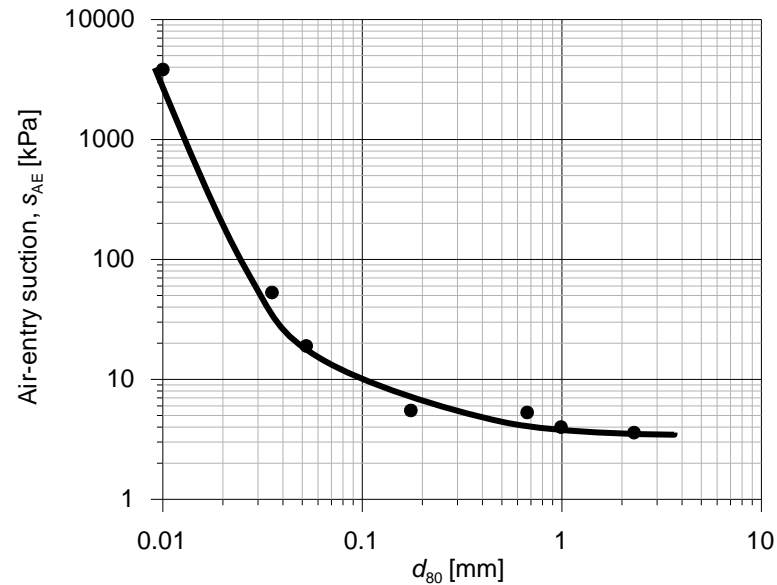
As a first approximation, the soil can be assumed saturated for suction lower than the air-entry value,  $s_{AE}$

The air-entry suction essentially depend on the pore size

For an order of magnitude if the air-entry suction :

<b>sand</b> <b>(<math>d_{\text{pore}}=30 \mu\text{m}</math>)</b>	<b>silt</b> <b>(<math>d_{\text{pore}}=3 \mu\text{m}</math>)</b>	<b>clay</b> <b>(<math>d_{\text{pore}}=0.3 \mu\text{m}</math>)</b>
<b>10 kPa</b>	<b>100 kPa</b>	<b>1000 kPa</b>

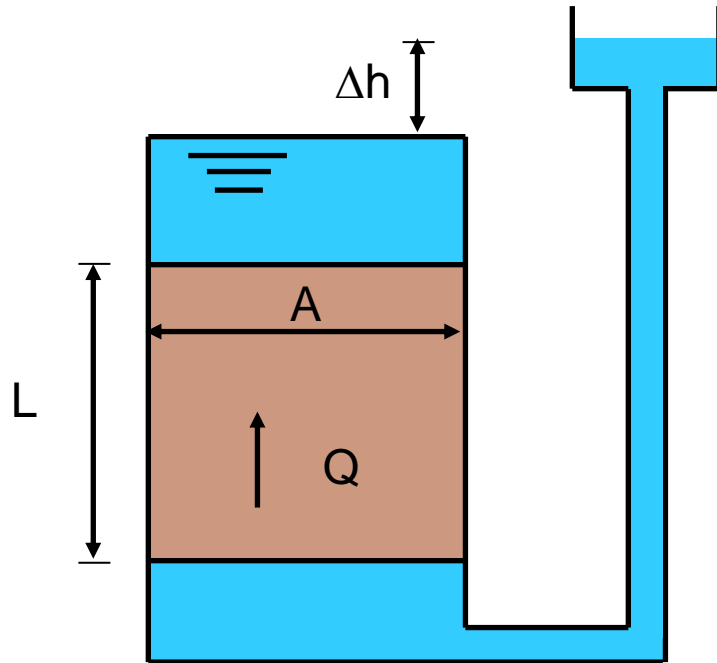
# Estimating the air-entry suction



Relationship between suction at air-entry,  $s_{AE}$ , and grain size  $d_{80}$ .

# Hydraulic conductivity behaviour

# Darcy's law (experiment)



$$v = \frac{Q}{A} = k \underbrace{\frac{\Delta h}{L}}_i = k \cdot i$$

$v$  = flow velocity

$Q$  = flow rate

$A$  = total area

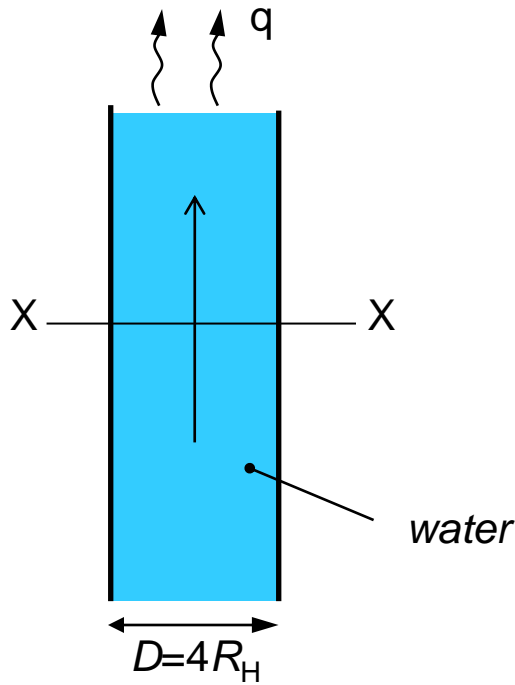
$k$  = hydraulic conductivity

$\Delta h$  = hydraulic head differential

$L$  = drainage length

# Flow through a **saturated** capillary tube

## *Poiseuille's law for laminar fluid flow*

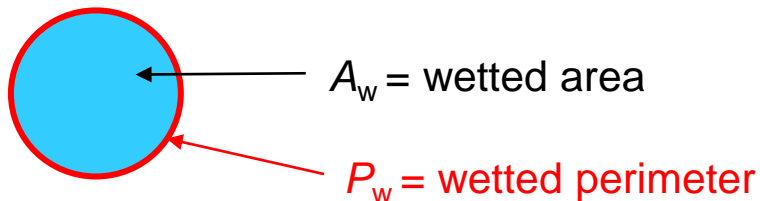


$$v_{act} = \frac{Q}{A_w} = \left[ \frac{g}{2\eta} R_H^2 \right] \cdot i$$

$Q$  = flow rate  
 $v_{act}$  = actual flow velocity  
 $i$  = hydraulic gradient  
 $A_w$  = wetted area  
 $\eta$  = kinematic viscosity  
 $\gamma$  = unit weight of the permeant  
 $R$  = tube radius  
 $R_H$  = hydraulic radius

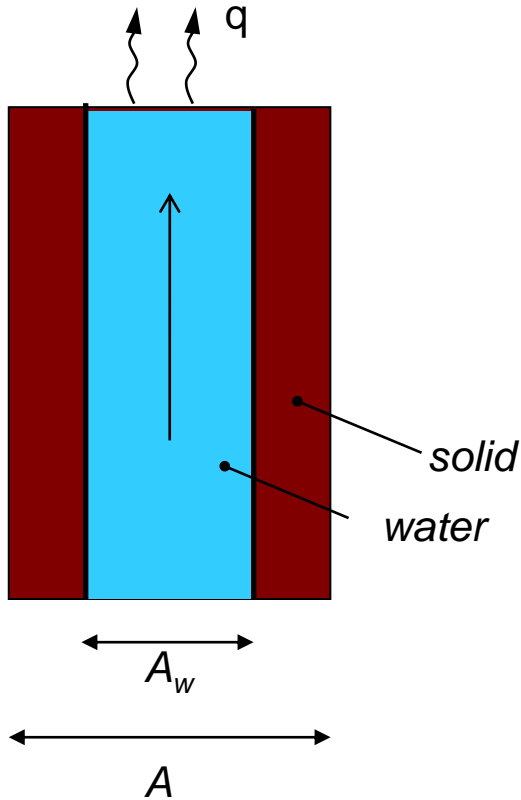
$$R_H = \frac{\text{flow channel cross section area}}{\text{wetted perimeter}} = \frac{A_w}{P_w} = \frac{\pi \frac{D^2}{4}}{\pi D} = \frac{D}{4}$$

section X-X





# Flow through a **saturated** capillary tube



$$v = \frac{Q}{A} = \frac{A_w}{A} \frac{Q}{A_w} = n \frac{Q}{A_w} \quad \left[ n = \frac{A_w}{A} = \text{porosity} \right]$$

$$v = \overbrace{\left[ n \frac{g}{2\eta} R_H^2 \right]}^k \cdot i$$

# Saturated hydraulic conductivity

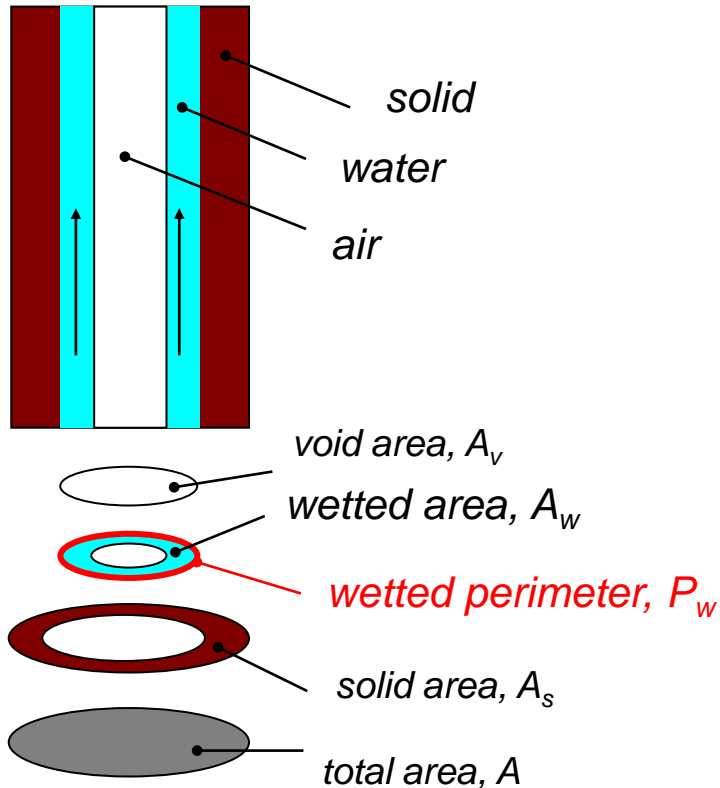
Hydraulic conductivity

$$k = n \frac{g}{32\eta} D^2 = n \frac{9.8 \frac{m}{s^2}}{32 \cdot 10^{-6} \frac{m^2}{s}} D^2 = n \left( 3 \cdot 10^5 \frac{1}{sm} \right) D^2$$

(Kinematic viscosity of water at 20°C  $\eta=10^{-6} \text{ m}^2/\text{s}$ )

	sand ↓	silt ↓	clay ↓	
$d_{\text{grain}} (\mu\text{m})$	300	30	3	
$d_{\text{pore}} (\mu\text{m})$	30	3	0.3	← $D$
$n \cdot k (m/s)$	$3 \cdot 10^{-4}$	$3 \cdot 10^{-6}$	$3 \cdot 10^{-9}$	

# Flow through an **unsaturated** capillary tube



$S_0$  = specific surface  
 $S_r$  = degree of saturation  
 $e$  = void ratio

As for the saturated tube:

$$A_w = S_r \frac{e}{1+e} A$$

$$P_w = A_s S_0 = \frac{1}{1+e} A S_0$$

$$v = \frac{v_{act} \cdot A_w}{A} = \left[ \frac{g}{2\eta} \frac{A_w^2}{P_w} \right] \cdot \frac{A_w}{A} \cdot i$$



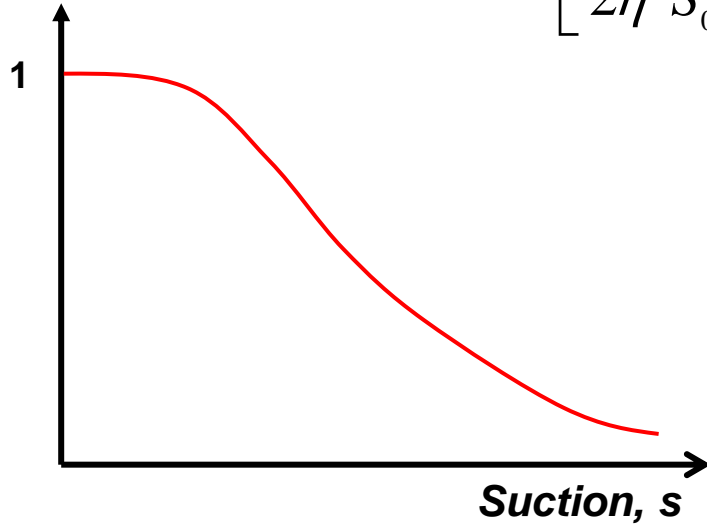
$$v = \underbrace{\left[ \frac{g}{2\eta} \frac{1}{S_0^2} \frac{e^3}{1+e} S_r^3 \right]}_k \cdot i$$

**Kozeny-Carman equation**

# Hydraulic conductivity of unsaturated soils

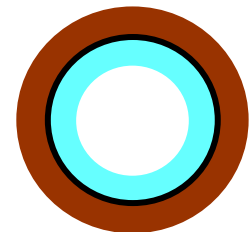
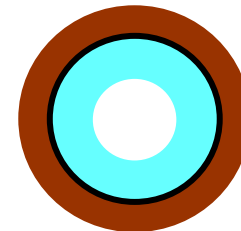
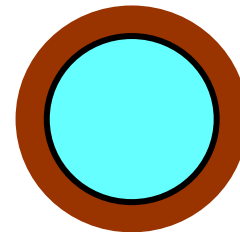
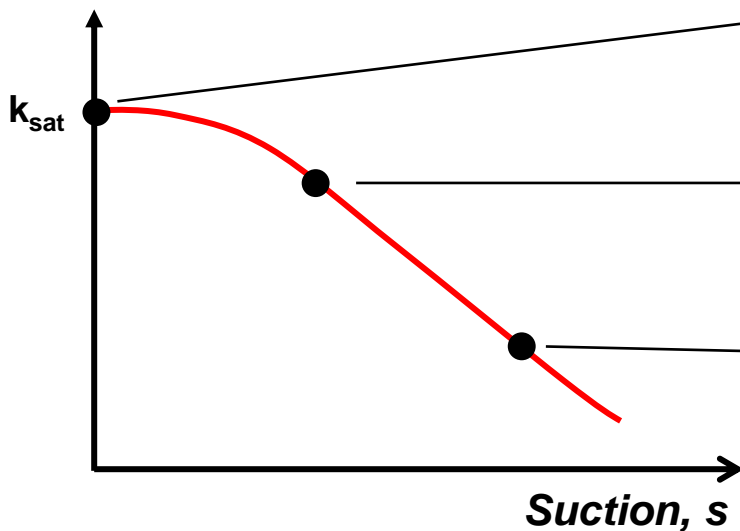
$$k = \left[ \frac{g}{2\eta} \frac{1}{S_0^2} \frac{e^3}{1+e} \right] S_r^3 = k_{sat} \cdot S_r^3$$

Degree of saturation,  $S_r$

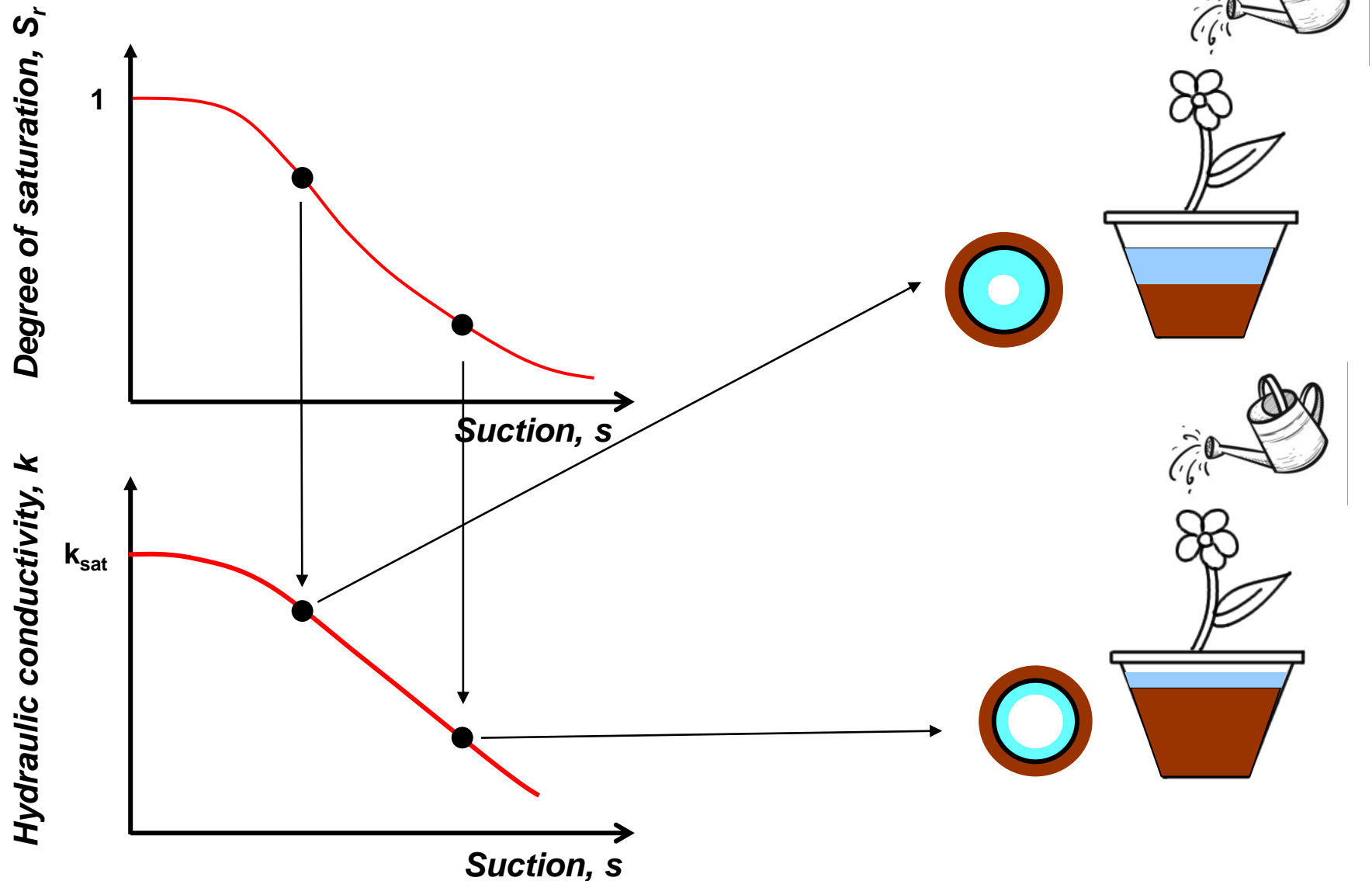


- Wetted Perimeter  $P_w$  remains constant
- Wetted Area  $A_w$  reduces

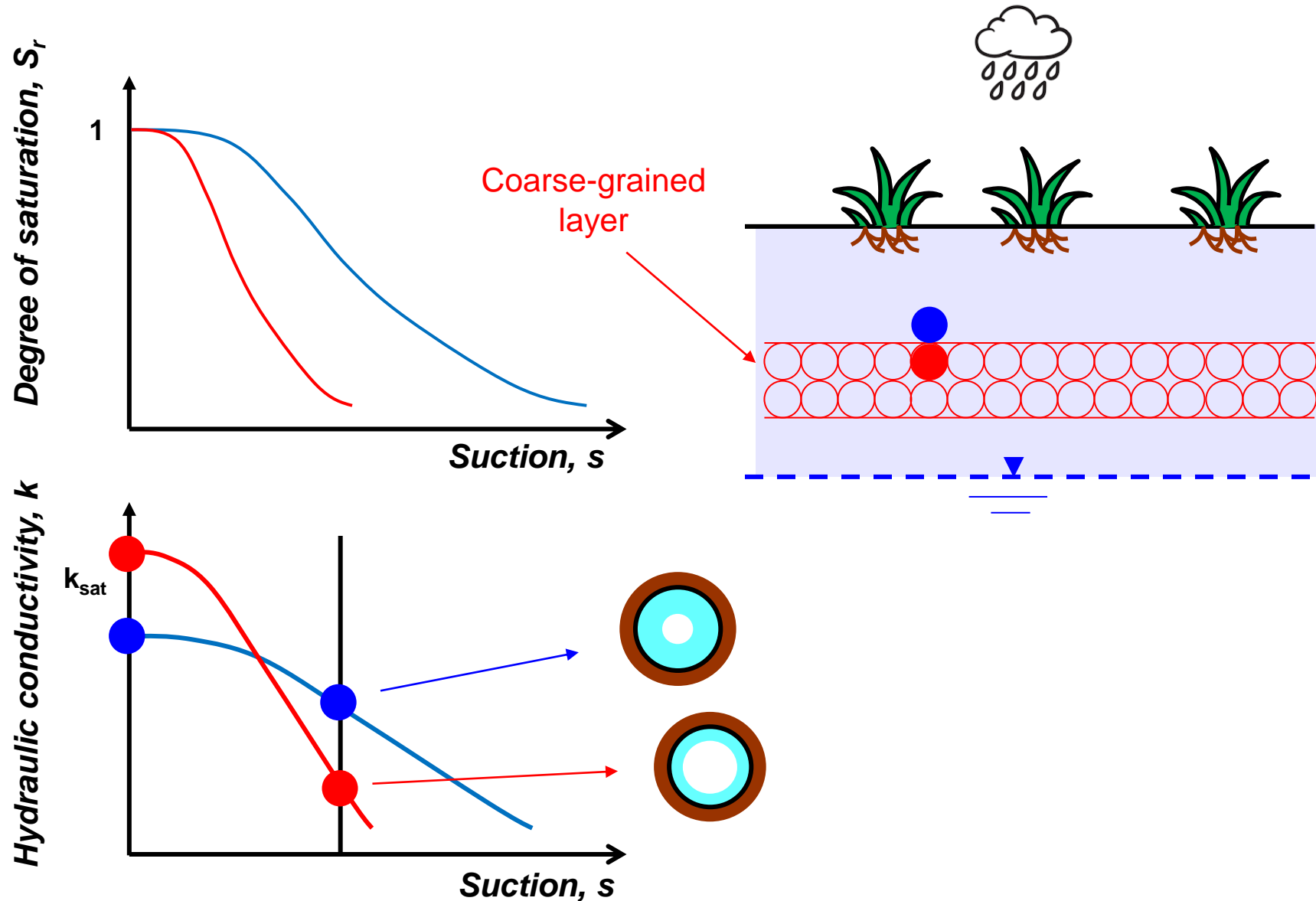
Hydraulic conductivity,  $k$



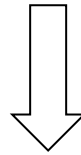
# Real life example of hydraulic conductivity behaviour of unsaturated soils



# Real life example of hydraulic conductivity behaviour of unsaturated soils: capillary barrier

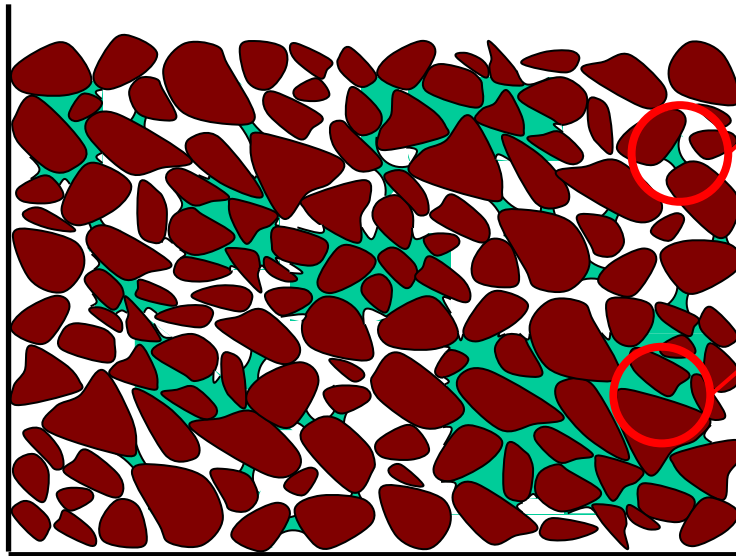


# ~~Micro-mechanical insight into shear strength of unsaturated geomaterials~~



*Tarantino A, Di Donna A (2019). Mechanics of unsaturated soils: simple approaches for routine engineering practice. Rivista Italiana di Geotecnica N. 4/2019*

# Bulk water and meniscus water

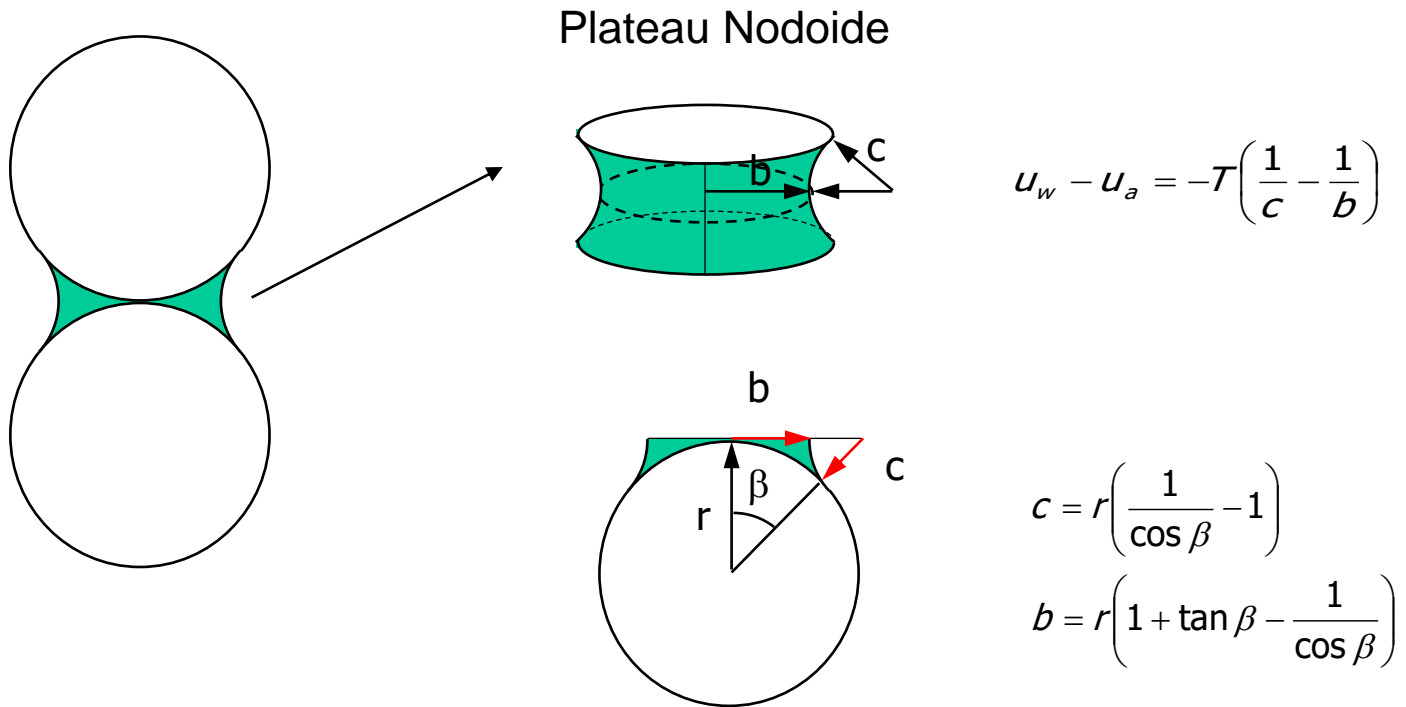


Meniscus water and meniscus contact

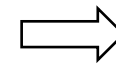
Bulk water and saturated contact



# Contact between spherical particles in presence of meniscus

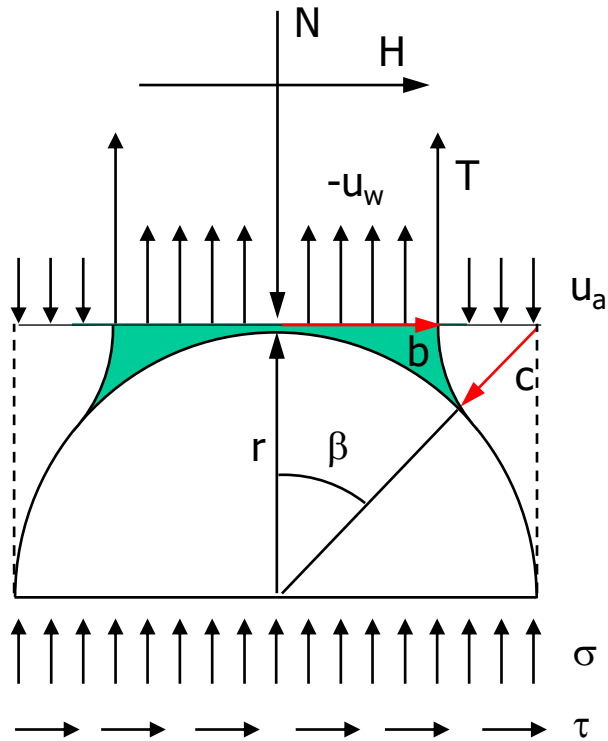


For  $b=53^\circ$ ,  $c=b$



$$u_w - u_a = 0$$

# Intergranular stress in presence of menisci



$N$  = normal intergranular force  
 $H$  = shear intergranular force  
 $u_w$  = water pressure  
 $u_a$  = air pressure  
 $T$  = surface tension

$P_w$  = wet perimeter  
 $A$  = totale area  
 $A_w$  = wet area  
 $\sigma$  = total normal stress  
 $\tau$  = shear stress

$$\sigma_i = \frac{N}{A} = (\sigma - u_a) + (u_a - u_w) \frac{A_w}{A} - T \frac{P_w}{A_w}$$



Total stress

Capillary water

$$\sigma_i = \frac{N}{A} = (\sigma - u_a) + \frac{1}{A} \left[ A_w + \frac{cb}{b-c} P_w \right] (u_a - u_w)$$

suction



$$\sigma_i = (\sigma - u_a) + \frac{1}{\pi r^2} \left[ \pi b^2 + \frac{cb}{b-c} 2\pi b \right] (u_a - u_w)$$

$f(\beta) \Rightarrow f(S_r)$

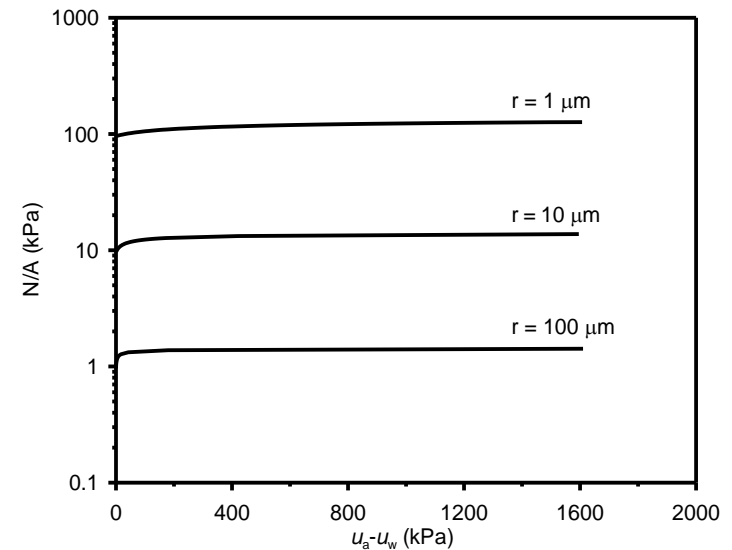
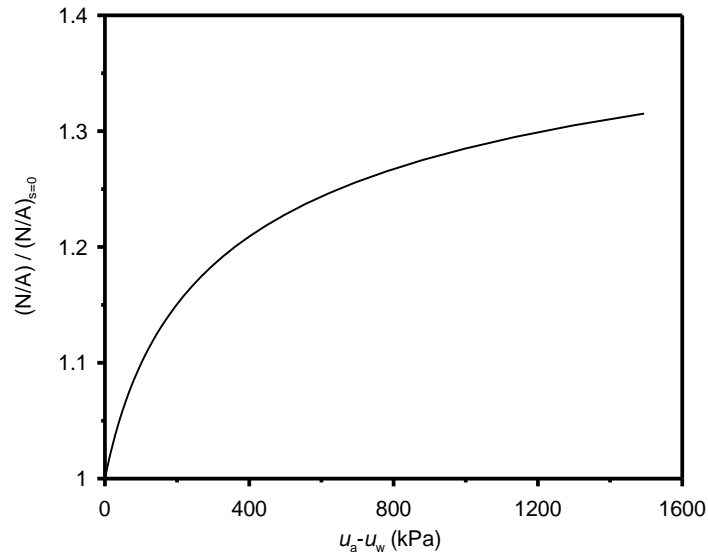


$$\sigma_i = (\sigma - u_a) + \frac{1}{\pi r^2} \left[ \pi b^2 + \frac{cb}{b-c} 2\pi b \right] (u_a - u_w)$$

See Tutorial T2c - Mechanical interactions at particle scale

# Effect of meniscus suction on intergranular stress

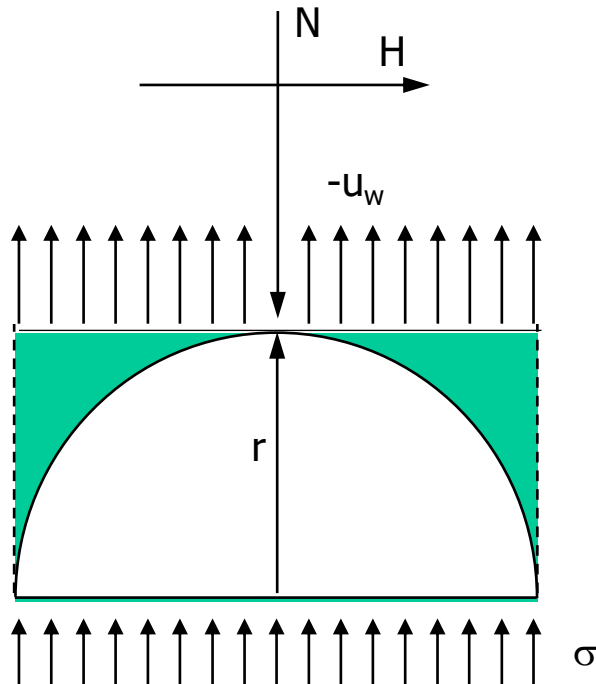
$$(\sigma_i)_m = \frac{N}{A} \Big|_{u_a - u_w} = \frac{1}{A} \left[ A_w + \frac{cb}{b-c} P_w \right] (u_a - u_w) = \frac{2T}{r} \left[ \frac{1}{1 + \tan\left(\frac{\beta}{2}\right)} \right]$$



The intergranular stress associated with meniscus suction undergoes little variation as suction increases.

As a first approximation, the intergranular stress generated by meniscus suction can be assumed to be constant

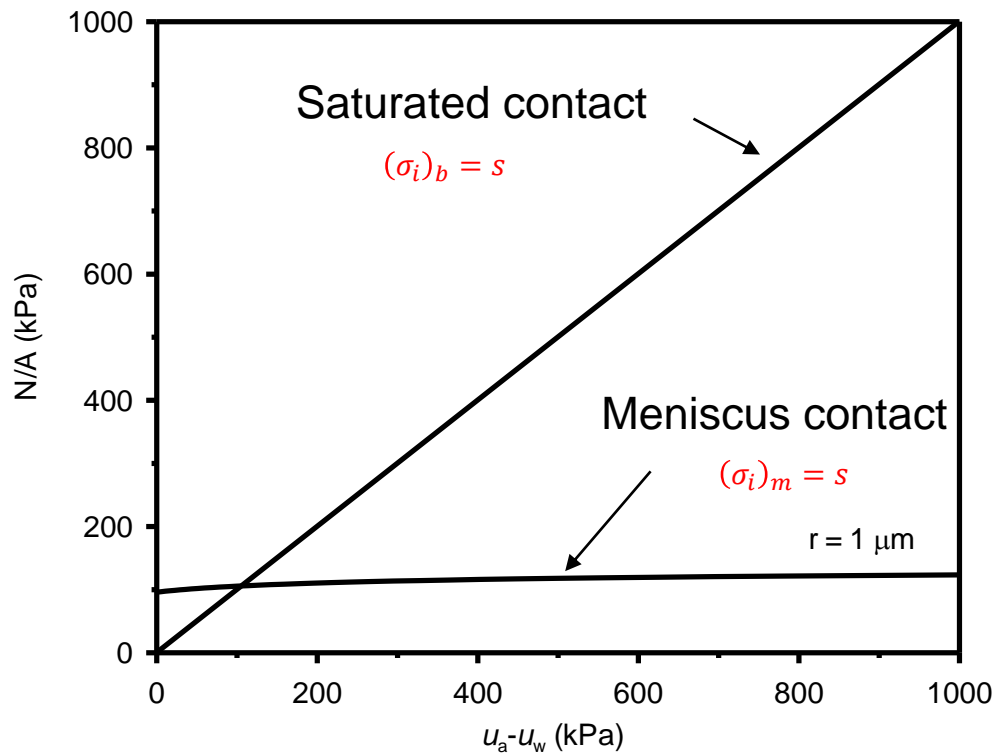
# Effect of bulk suction on intergranular stress



$$\sigma_i = \frac{N}{A} = \sigma - u_w = \sigma + s$$

# Effect of suction on intergranular stress for saturated contact

$$\left. \frac{N}{A} \right|_{u_a - u_w} = -u_w$$



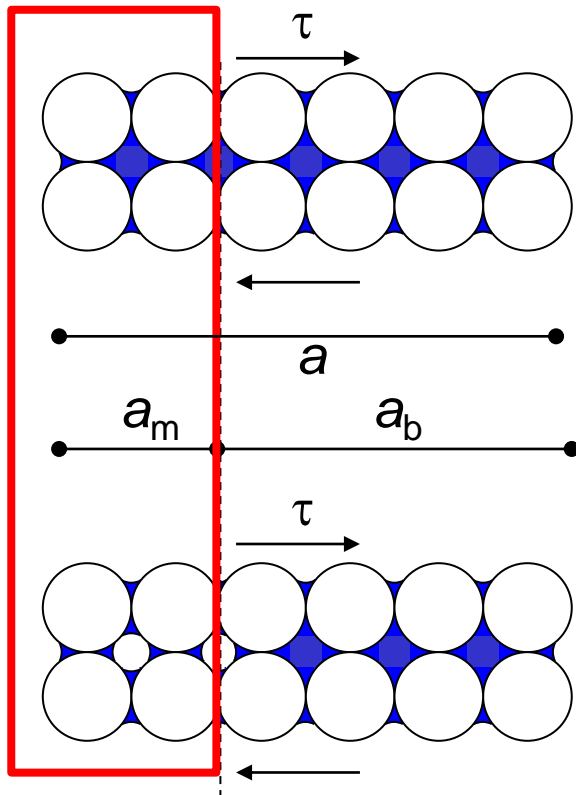
For saturated contact, the intergranular stress changes linearly with suction

# Effect of suction on shear strength

ASSUMPTION

$$\tau = \sigma_i \cdot \tan \phi'$$

This may be acceptable only from a qualitative point of view !!



$$(\sigma_i)_s = (\sigma_i)_b = s$$



$$(\sigma_i)_s = s \frac{a_m}{a} + s \frac{a_b}{a}$$

**Saturated**

$$(\sigma_i)_s = (\sigma_i)_m \frac{a_m}{a} + (\sigma_i)_b \frac{a_b}{a}$$

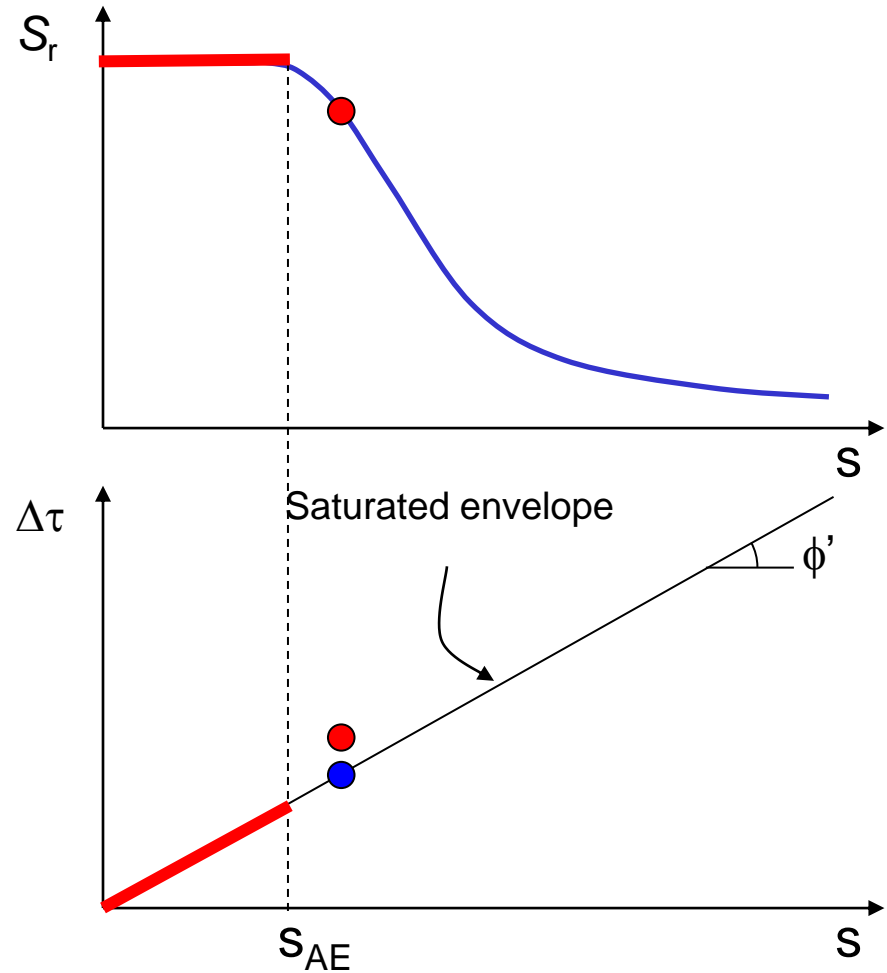
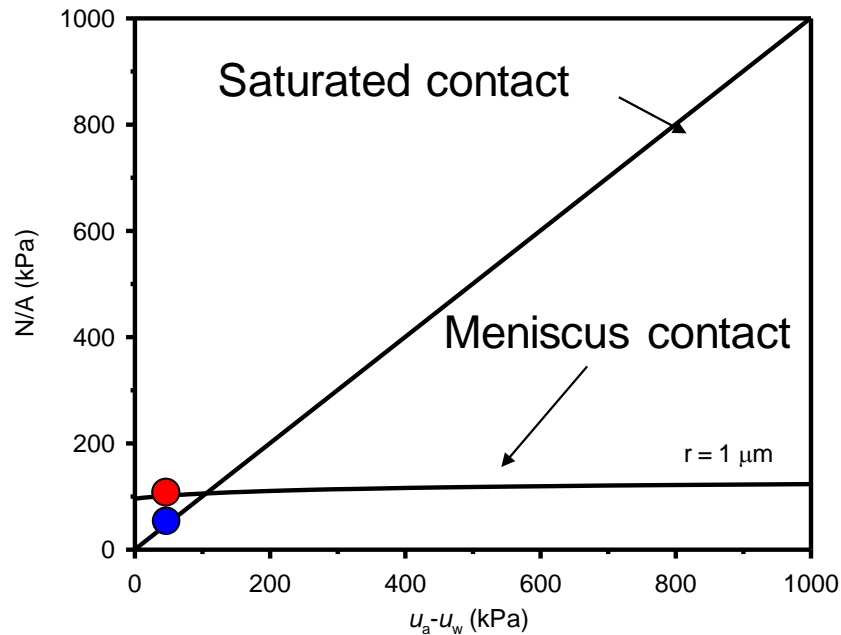
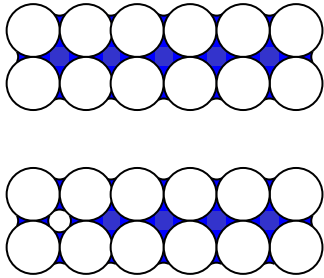


$$(\sigma_i)_s = (\sigma_i)_m \frac{a_m}{a} + s \frac{a_b}{a}$$

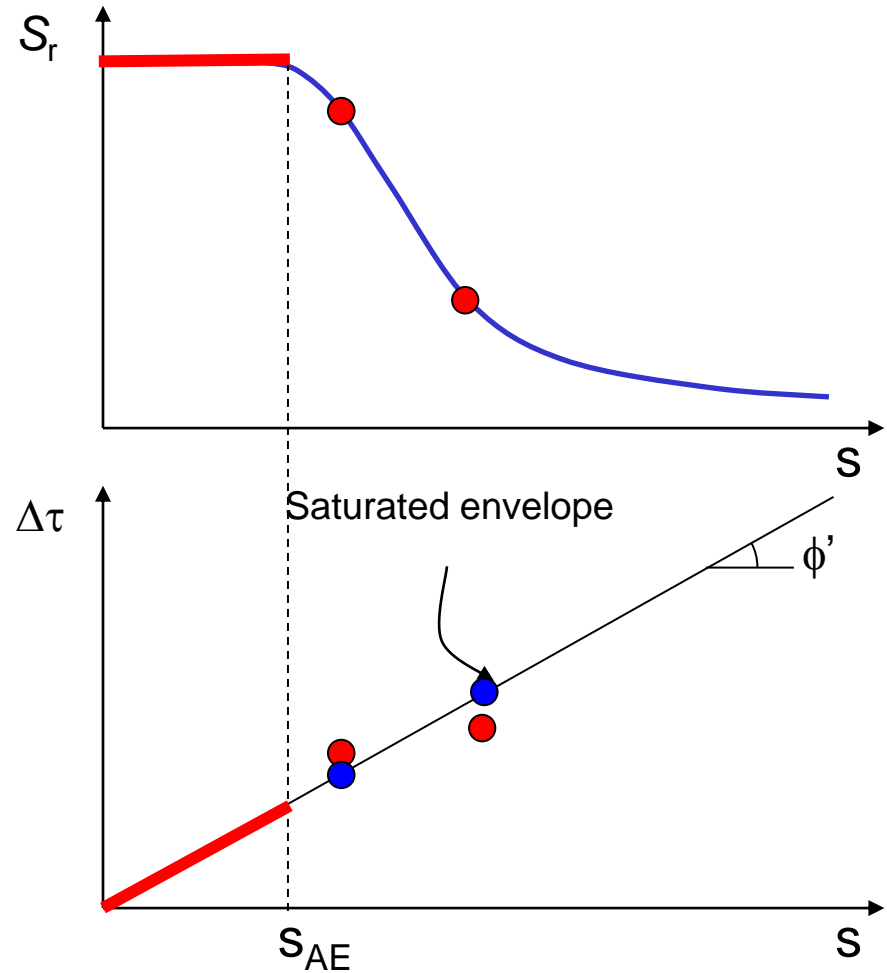
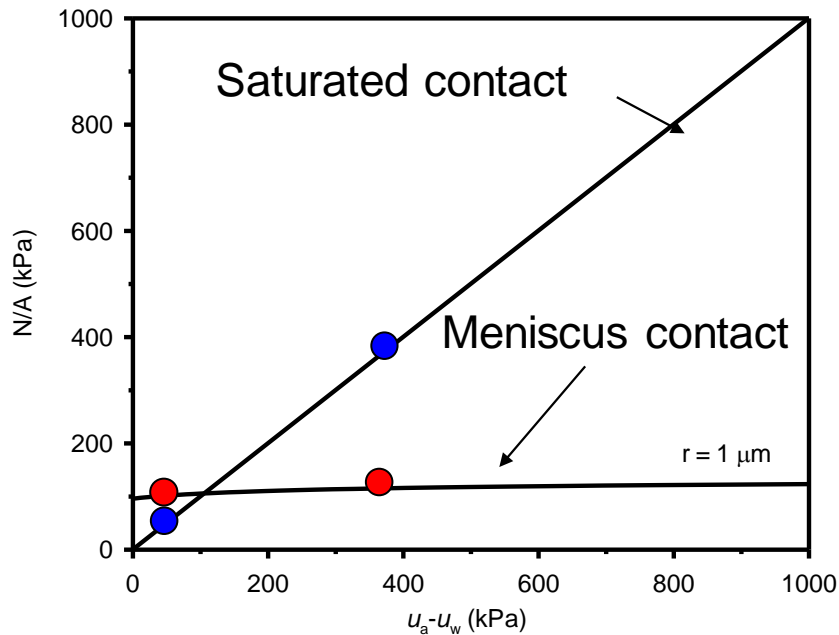
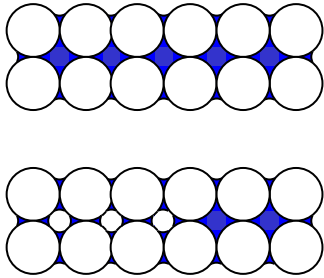
**Partially  
saturated**

# Shear strength of unsaturated soils:

## *Quasi-saturated state*

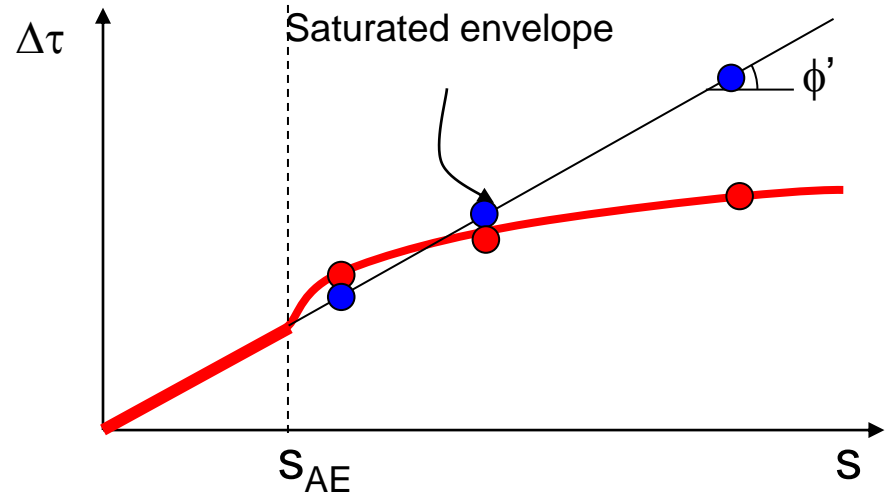
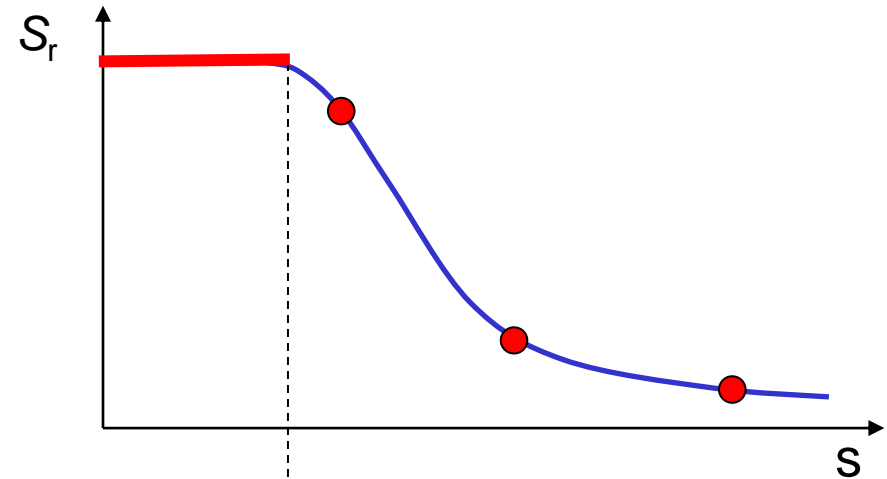
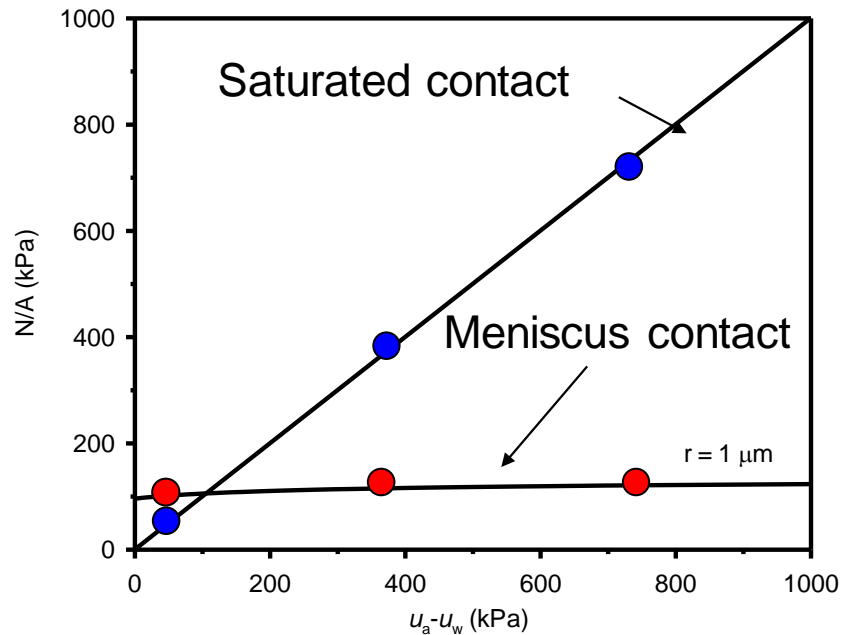
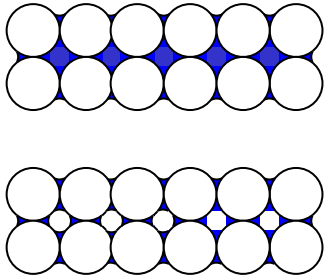


# Shear strength of unsaturated soils: Partially saturated state

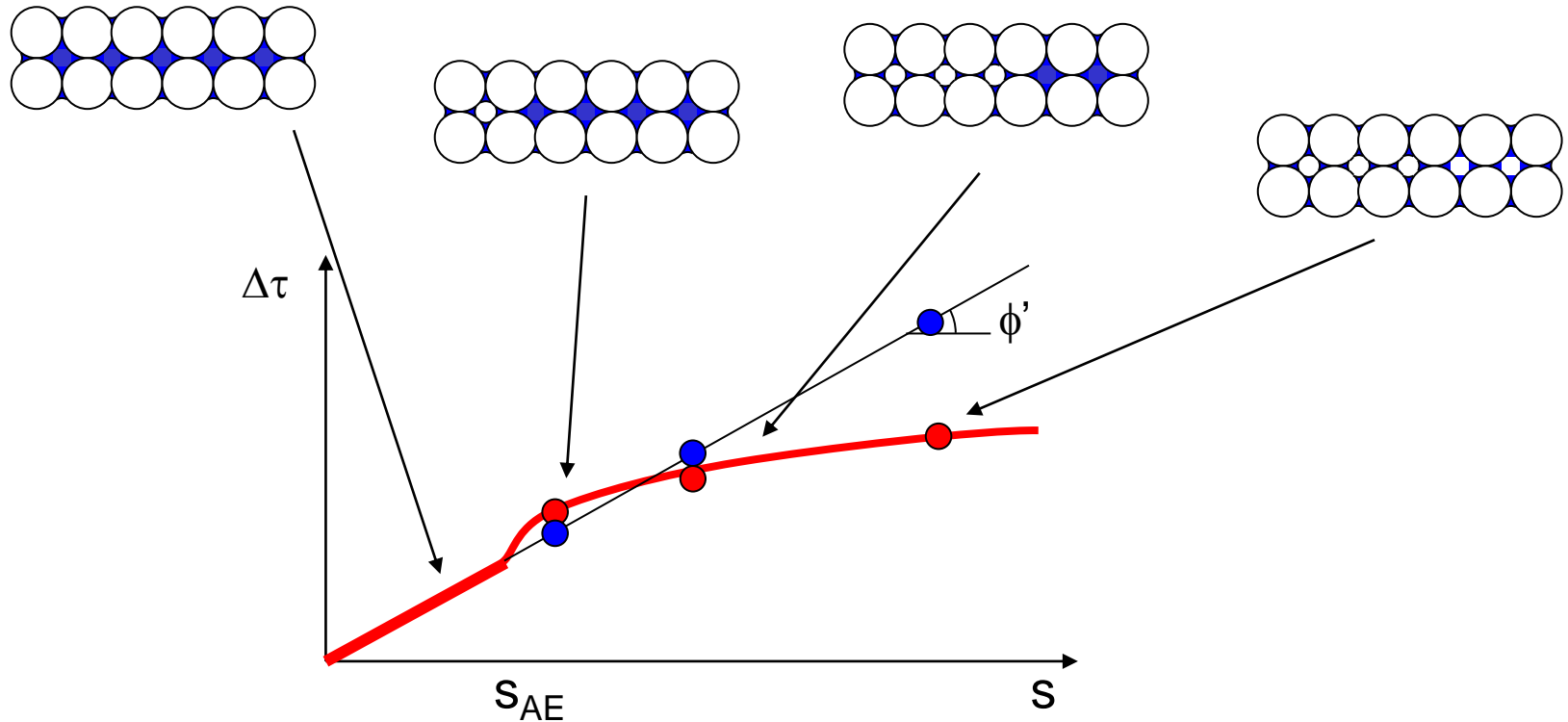




# Shear strength of unsaturated soils: Residual state



# Shear strength envelope

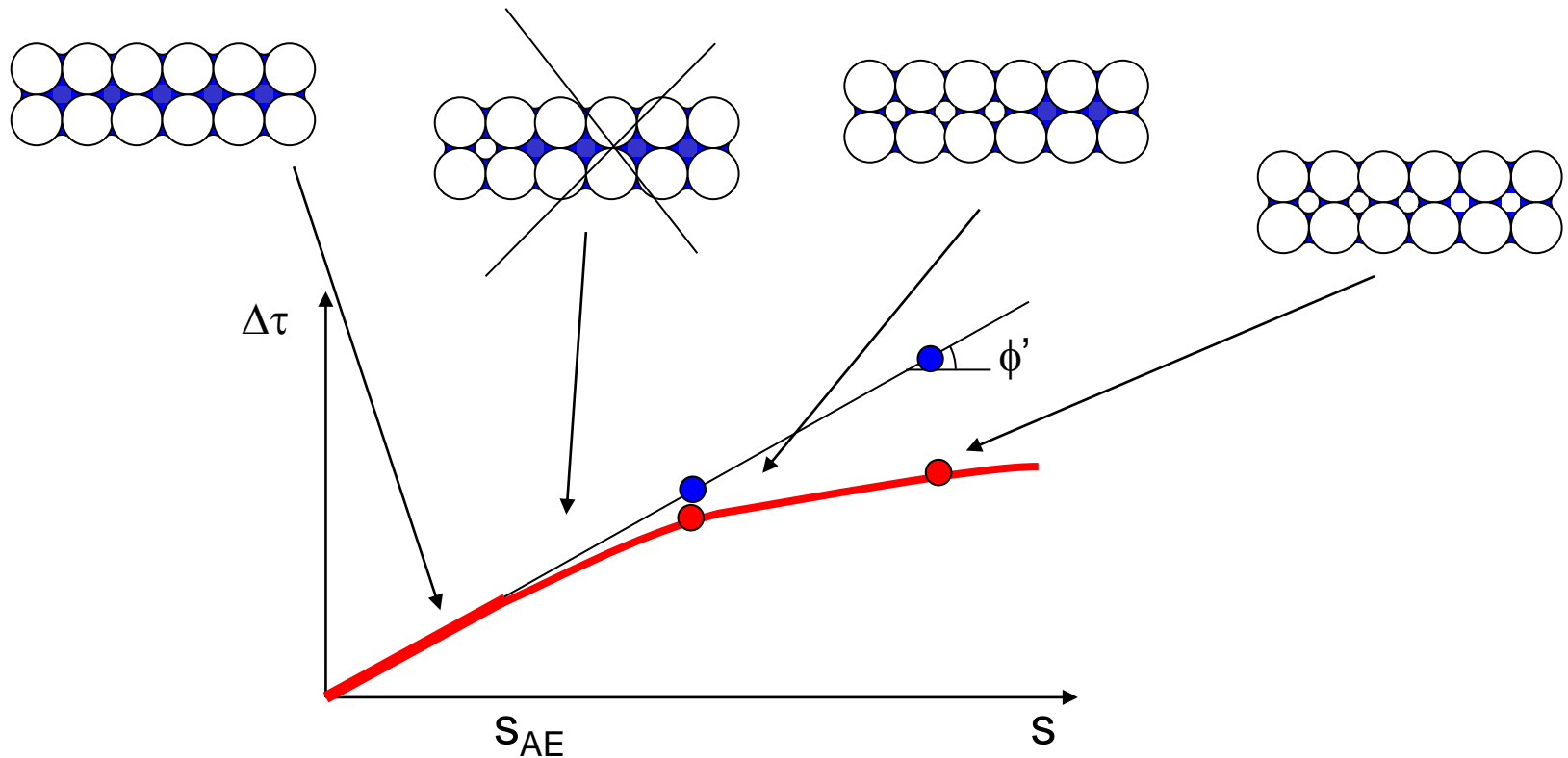


Shear strength depends on suction (controlling the intergranular stress in the bulk water) and on the number of menisci, in turn controlled by the degree of saturation

$$\Delta\tau = \Delta\tau (s, S_r)$$

# Shear strength envelope

Not always observed experimentally



Shear strength depends on suction (controlling the intergranular stress in the bulk water) and on the number of menisci, in turn controlled by the degree of saturation

$$\Delta\tau = \Delta\tau (s, S_r)$$

# **Shear strength behaviour**

# Shear strength of saturated soils

## *Saturated state*

$$\tau = (\sigma - u_w) \tan \phi'$$



$$\tau = (\sigma + s) \tan \phi'$$

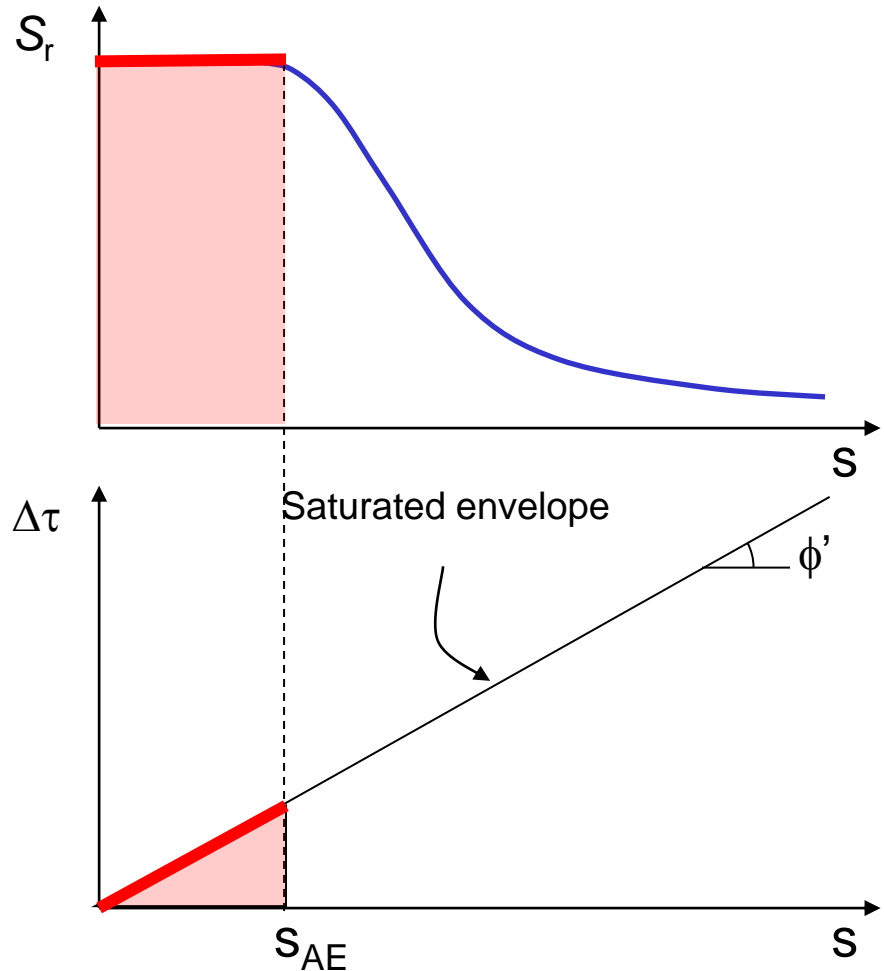


$$\tau = \sigma \tan \phi' + s \tan \phi'$$



$$\tau = \sigma \tan \phi' + \Delta\tau(s)$$

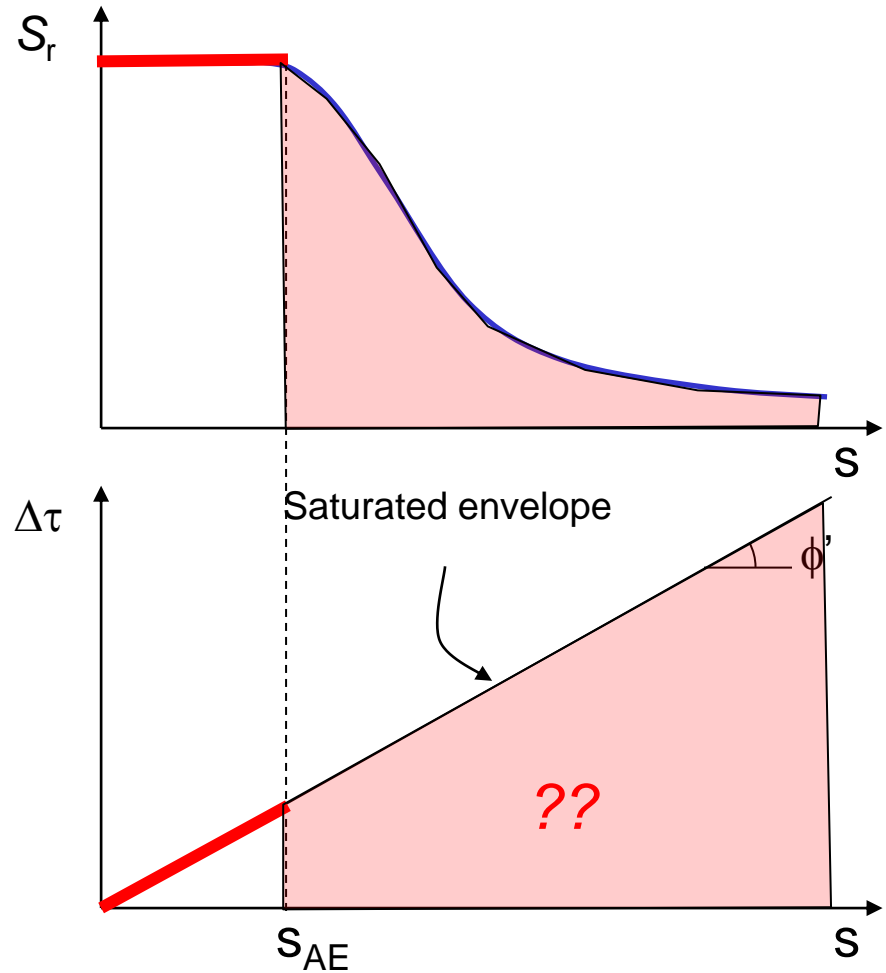
$$s \tan \phi'$$



# Shear strength of unsaturated soils

$$\tau = \sigma \tan \phi' + \Delta\tau(s)$$

↓  
??

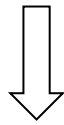


# Shear strength criteria for sandy/silty unsaturated soils

## Sandy/Silty soils

$$\Delta\tau_{suction} = s \cdot \tan\phi' \quad s \leq s_{AE}$$

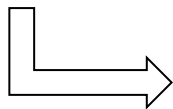
$$\Delta\tau_{suction} = (s \cdot S_r) \cdot \tan\phi' \quad s > s_{AE}$$



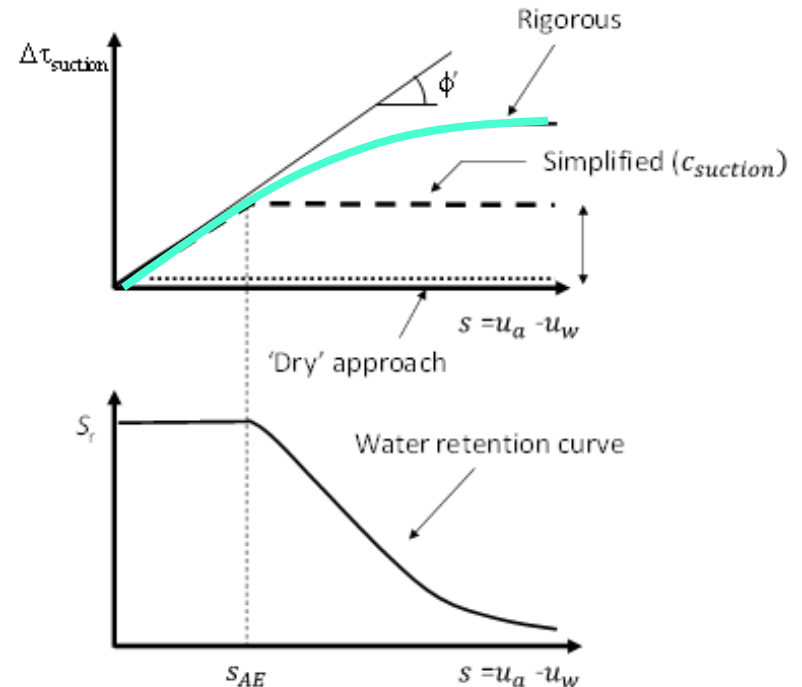
$$\tau = \sigma \cdot \tan\phi' + \frac{\Delta\tau_{suction} = c_{suction}}{(s \cdot S_r) \cdot \tan\phi'}$$

$$\tau = \underbrace{(\sigma + s \cdot S_r)}_{\text{Unsaturated effective stress}} \cdot \tan\phi'$$

$$\tau = \underbrace{(\sigma - u_w \cdot S_r)}_{\text{Unsaturated effective stress}} \cdot \tan\phi'$$



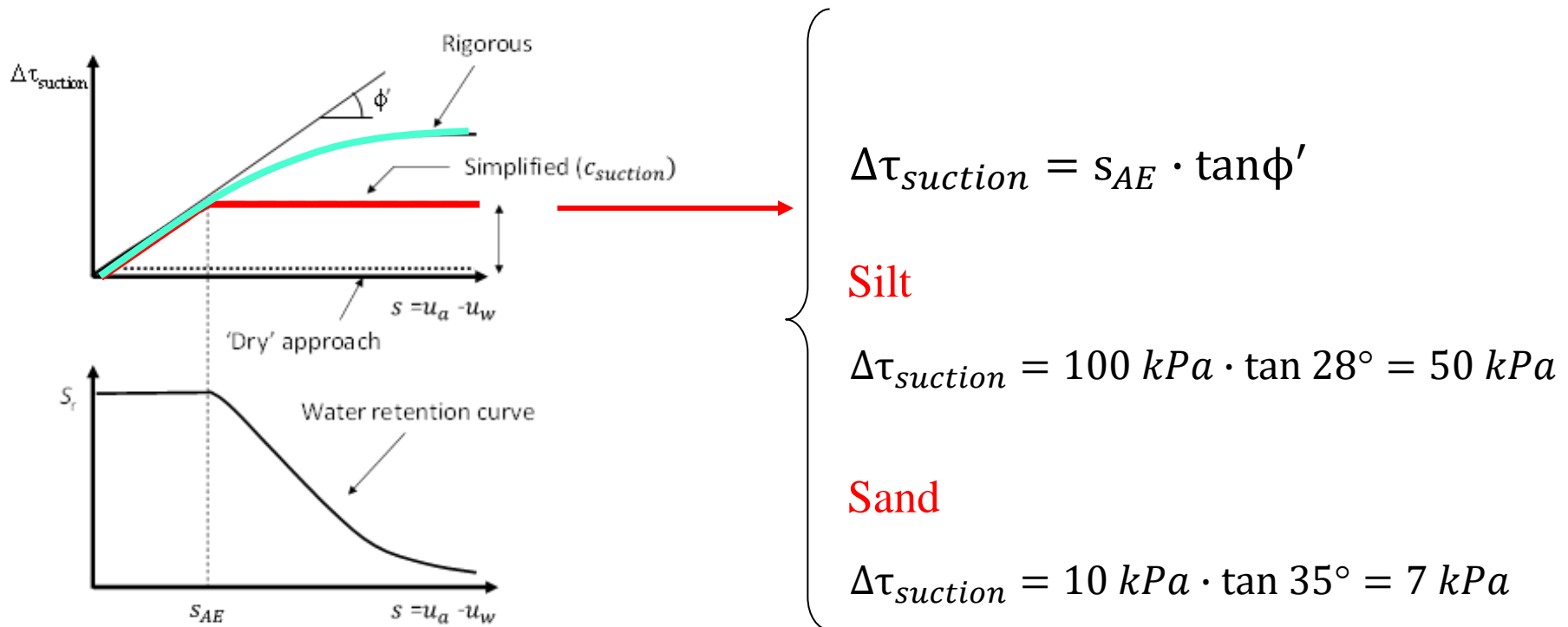
- Need for the relationship between suction  $s$  and degree of saturation  $S_r$ , i.e. water retention curve
- Need to predict evolution of suction  $s$  and degree of saturation  $S_r$  in response to rainfall, i.e. water retention curve and hydraulic conductivity



# Simplified shear strength criterion for geotechnical applications

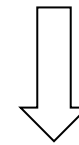
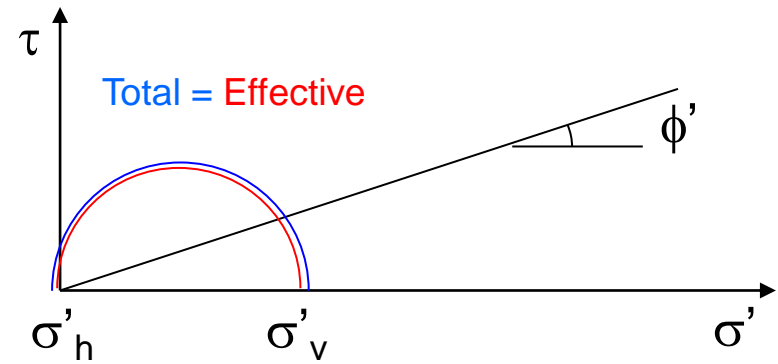
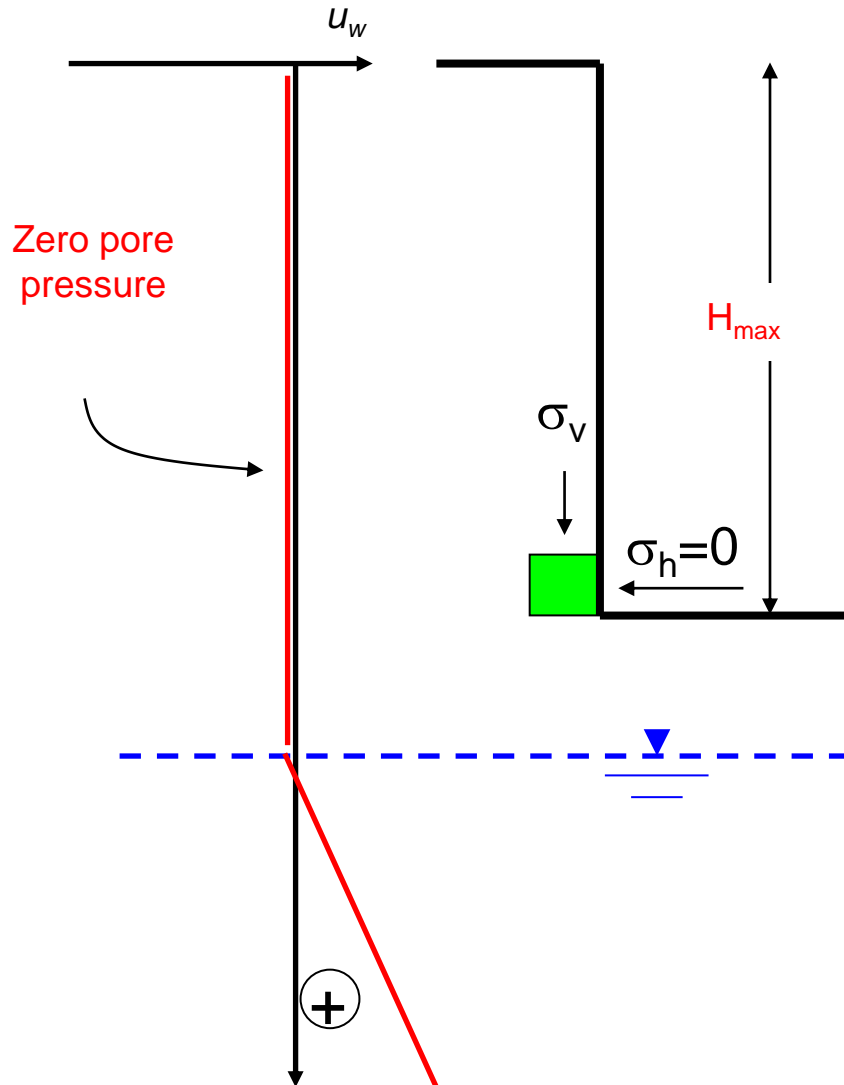
$$\Delta\tau_{suction} = s \cdot \tan\phi' \quad s \leq s_{AE}$$

$$\Delta\tau_{suction} = s_{AE} \cdot \tan\phi' \quad s > s_{AE}$$





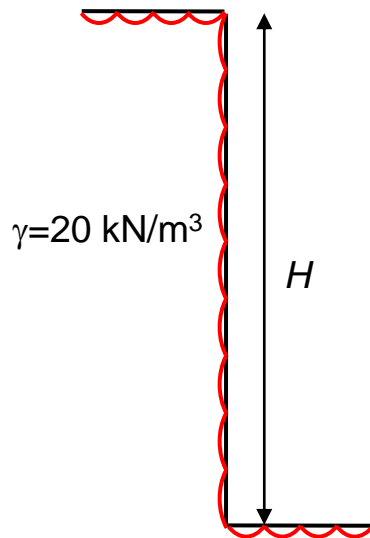
# Stability of vertical cut in silt: 'dry' approach



$$H_{\max} = 0$$

Vertical cuts cannot be stable

# Stability of vertical cut: unsaturated 'simplified' approach



In the classical dry soil mechanics,  $H=0$  if  $c'=0$

Very little evaporation is sufficient to curve menisci and lower water pressure to the air entry value, say  $u_w=-100$  kPa

As the soil is saturated, the shear strength criterion can be written as follows:

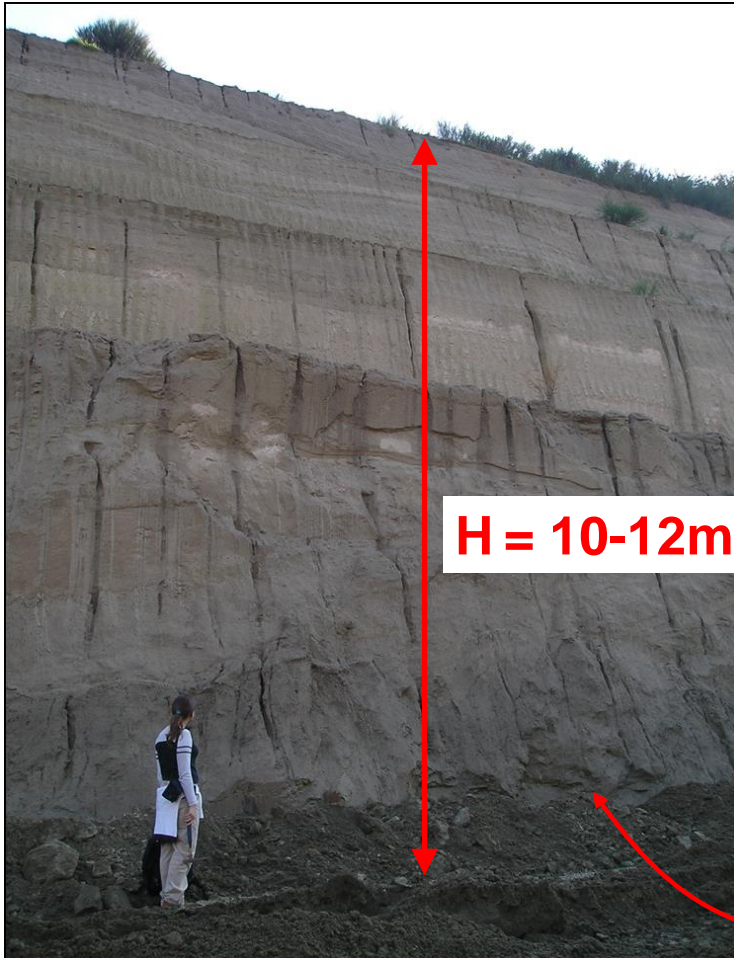
$$\tau = (\sigma - u_w) \tan \phi' = \sigma \tan \phi' + (-u_w) \tan \phi' = \sigma \tan \phi' + c_{\text{apparent}}$$

Assuming  $\phi'=28^\circ$ , risulta  $c_{\text{apparent}} = 50$  kPa

Assuming that  $H=2c / \gamma(k_a)^{1/2}$  we obtain  $H= 7.8$  m!

# Stable vertical cuts in 'cohesionless' soils

(De Vita et al. 2008, IJEGE)



**H = 10-12m**

**Dry approach ( $c'=0$ )**

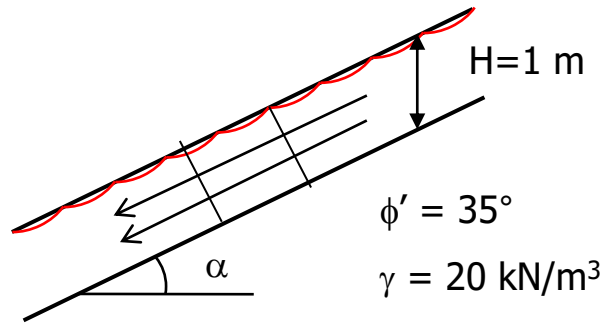


**$H_{\max} = 0$  !!!**

Pyroclastic 'cohesionless' silty sand

Giugliano near Naples, Italy  
(courtesy of Prof. De Vita, University of Naples Federico II)

# Infinite slope in sand



Water table at the ground surface.

$$\eta = \left(1 - \frac{\gamma_w}{\gamma}\right) \frac{\tan \phi'}{\tan \alpha} = 1 \quad \Rightarrow \quad \alpha_{\max} = 19^\circ$$

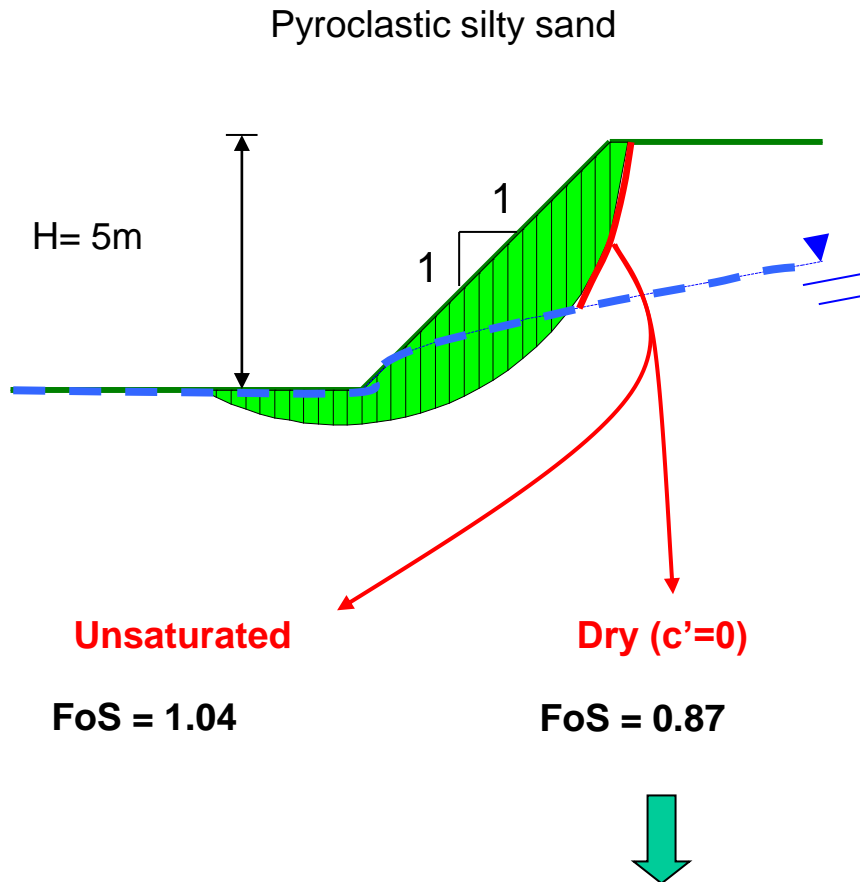
Very little evaporation is sufficient to curve menisci and lower water pressure to the air entry value, say  $u_w = -10 \text{ kPa}$

As the soil is saturated, the factor of safety can be written as follows:

$$\eta = \left(1 - \frac{\gamma_w}{\gamma}\right) \frac{\tan \phi'}{\tan \alpha} + \frac{(-u_w) \tan \phi'}{H \gamma \sin \alpha \cos \alpha}$$

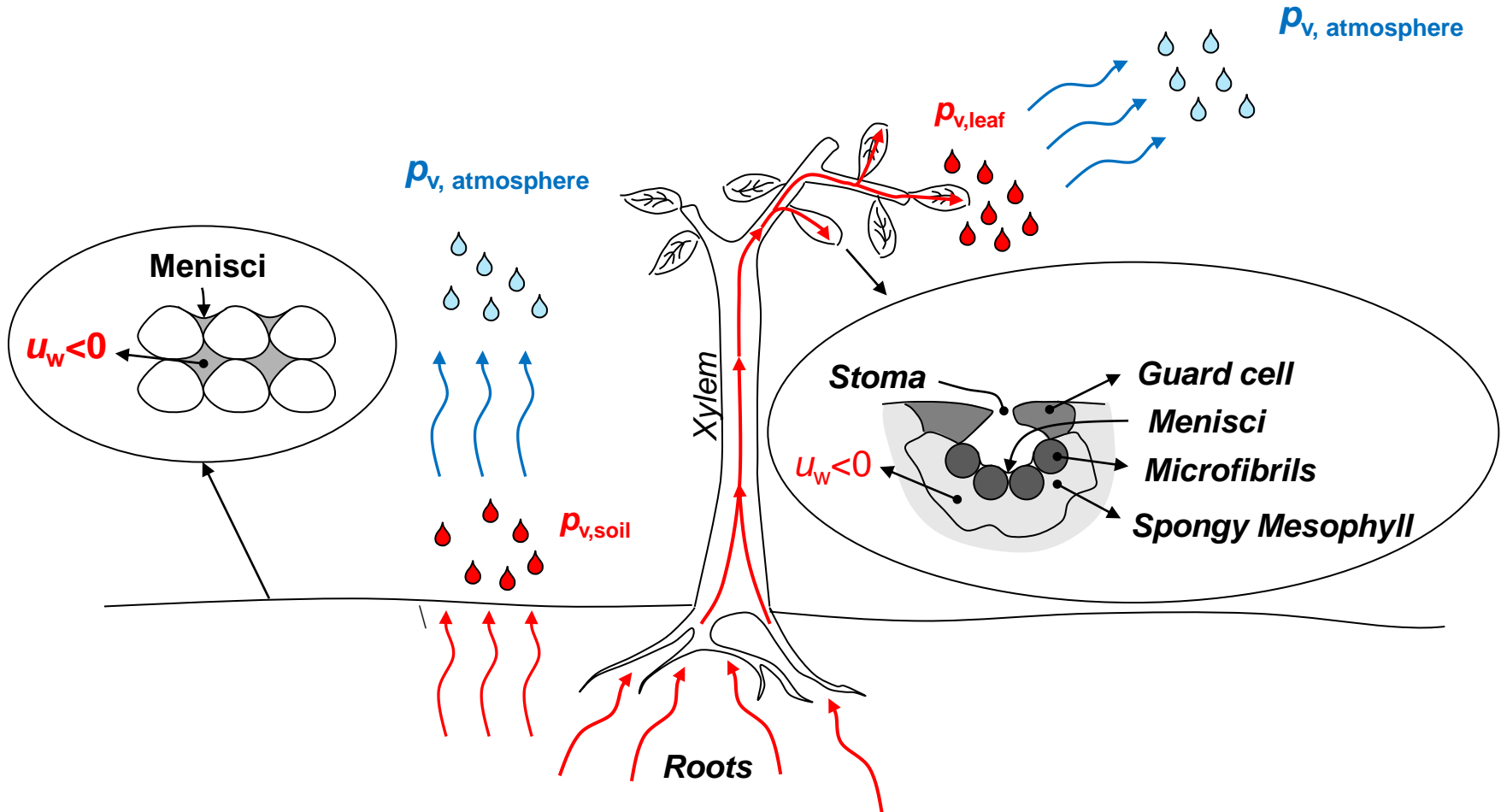
Assuming  $\eta = 1$ , we obtain  $\alpha_{\max} = 50^\circ$  !

# Stable cut slope in 'cohesionless' soils



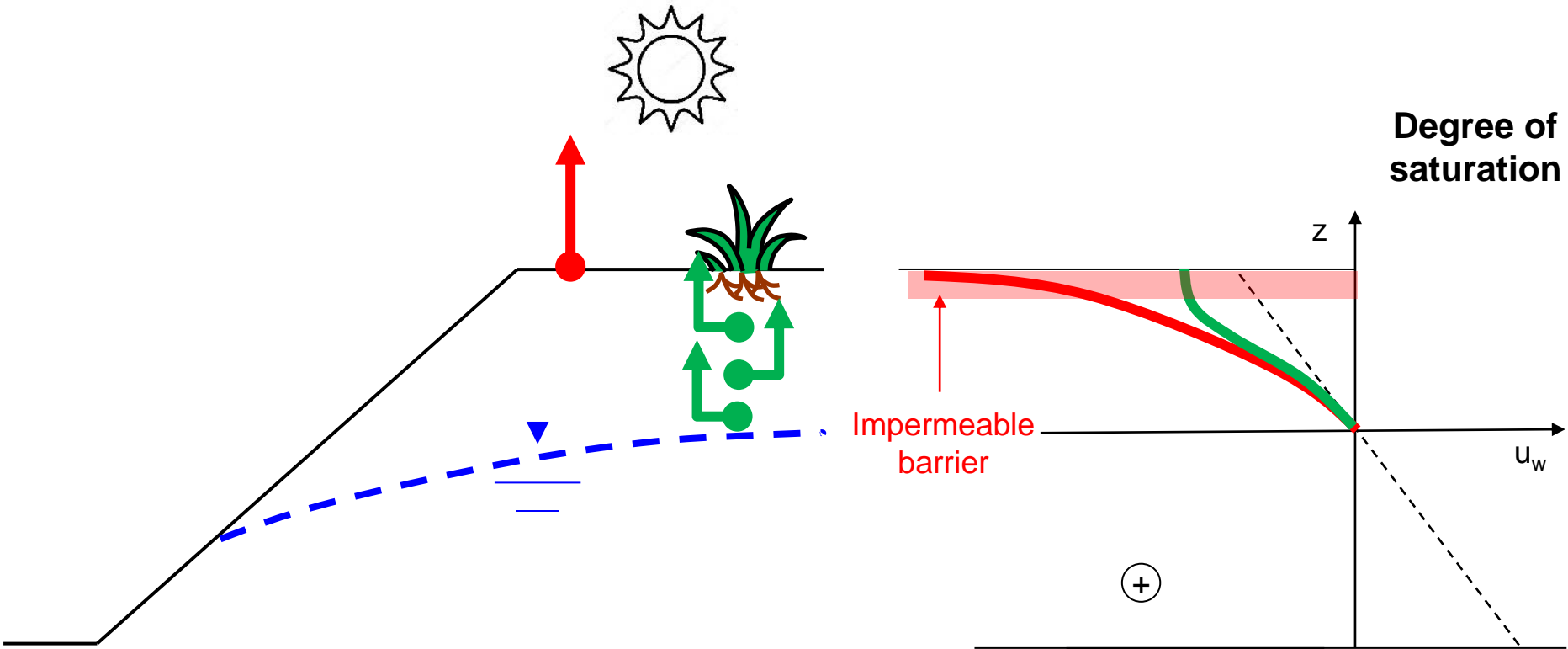
**Remedial measures:  
thinking out loud**

# Driving mechanism of evaporation and transpiration



- Water is extracted by the **atmosphere** THROUGH the plant (and not by the plant)
- The driving mechanism of water extraction is the same for **bare** and **vegetated** soil

# Remove soil water by transpiration (but not evaporation)

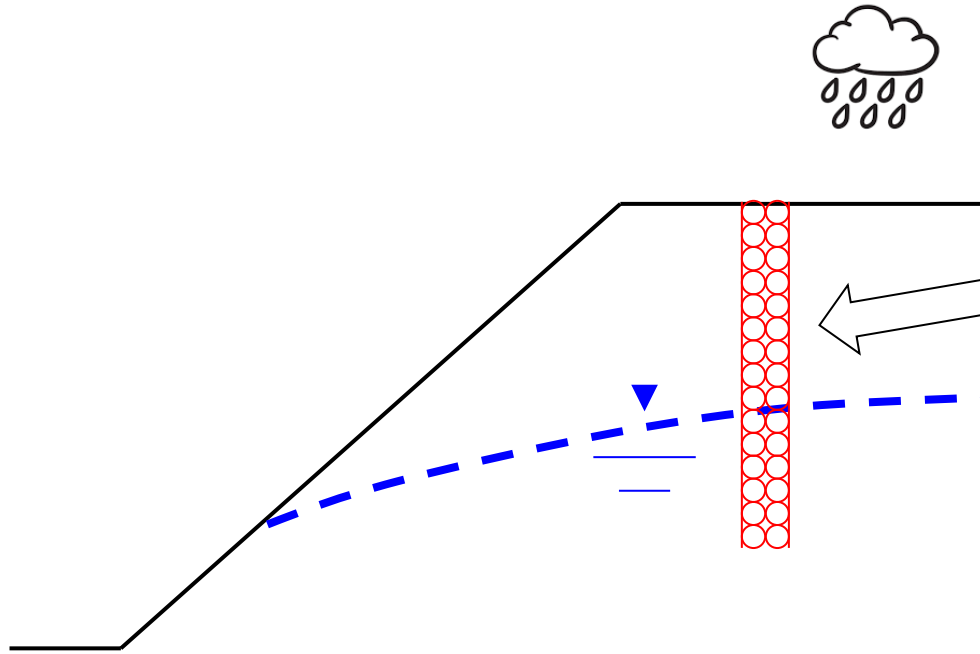


## Transpiration

Gentler suction generate lower shear strains minimising susceptibility to cracking and maintaining unsaturated soil more conductive

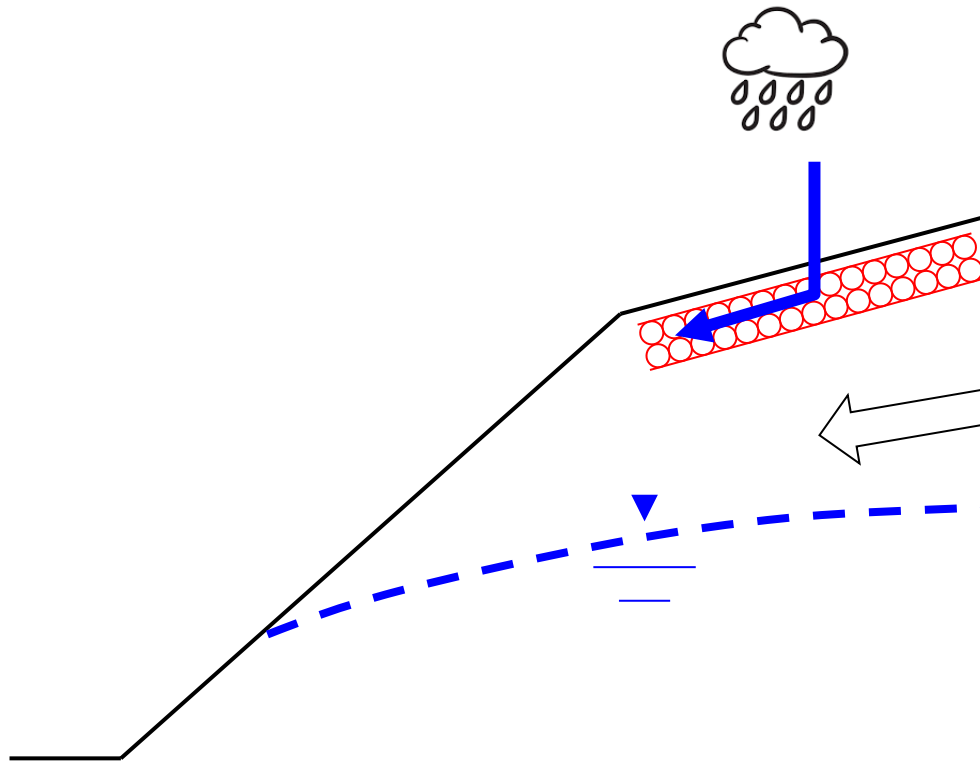


# No coarse-grained drains above phreatic surface



Coarse-grained layers act as capillary barrier above phreatic surface and favour water accumulation

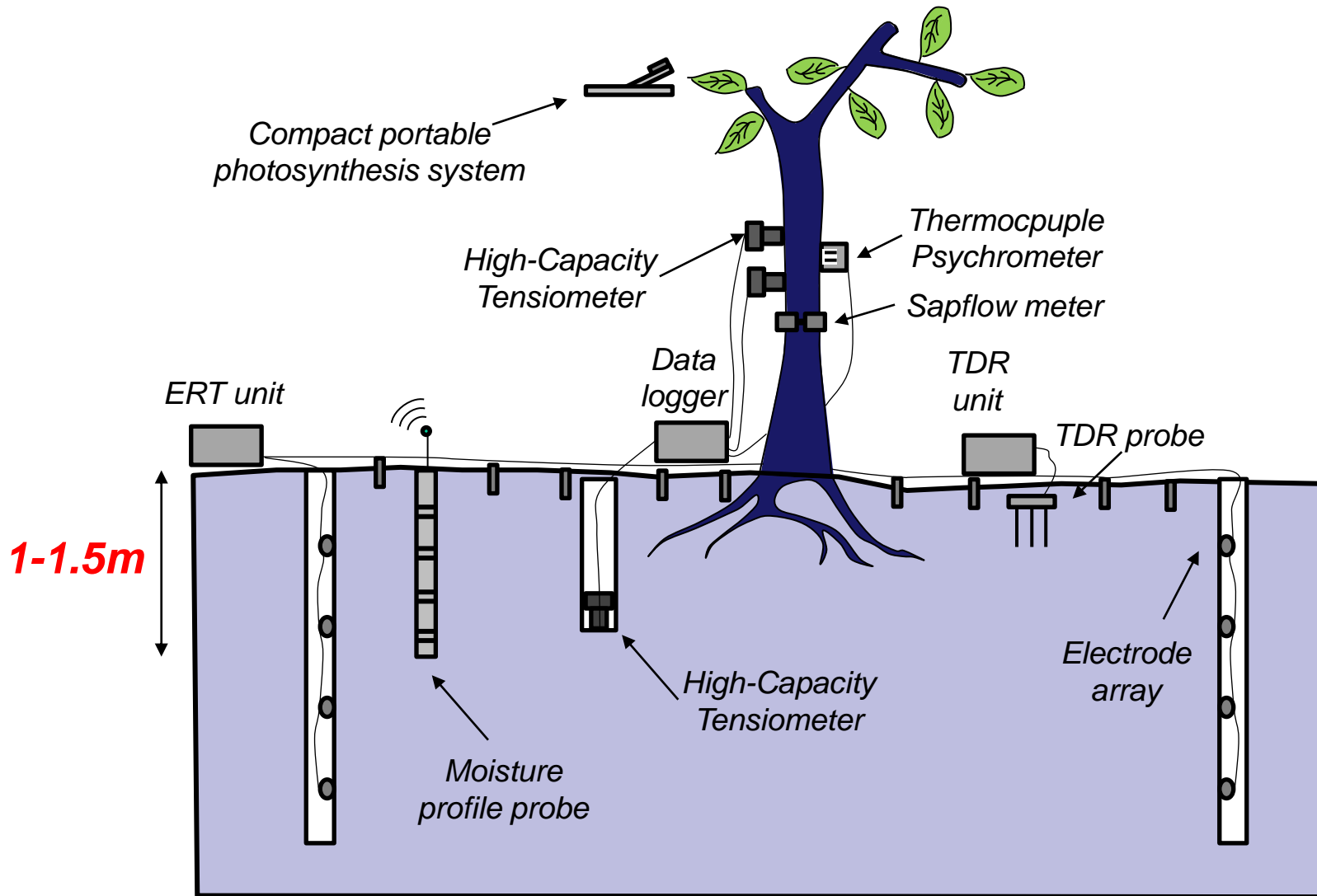
# Divert rainwater



Coarse-grained surface layers can act as natural drainage

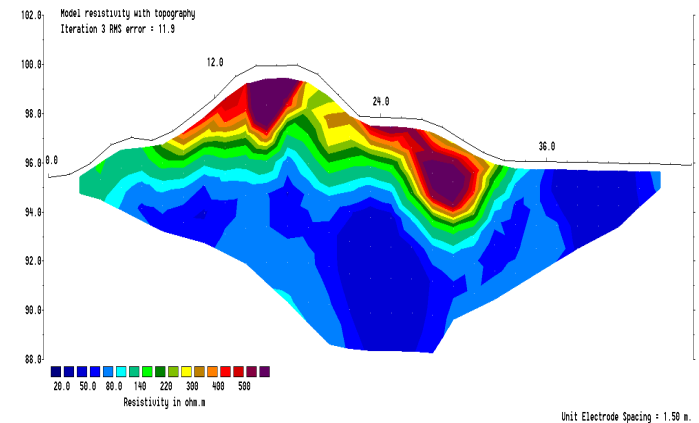
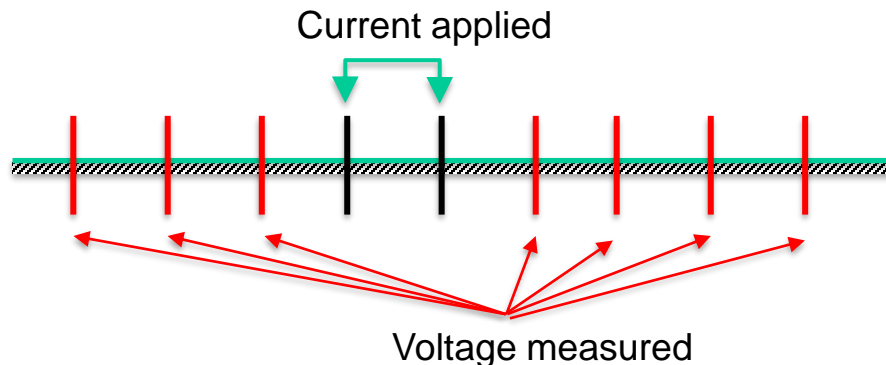
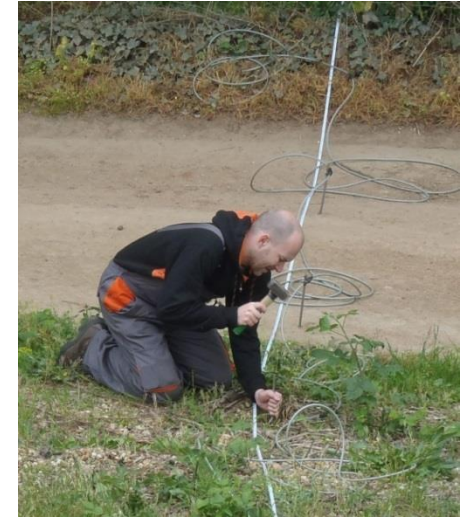
# **Monitoring the vadose zone**

# Monitoring borrowed from agricultural systems

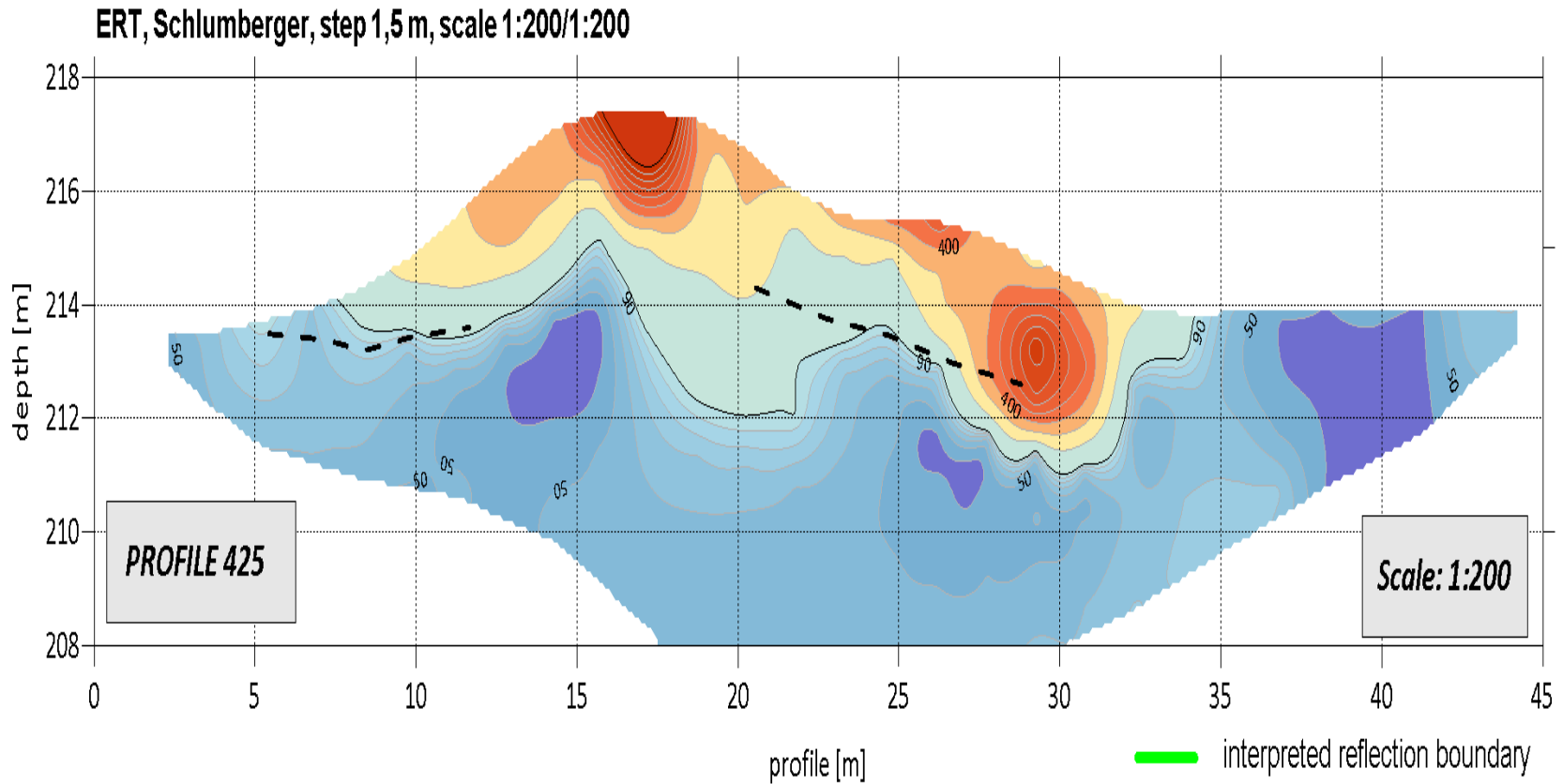


# ERT - Electrical Resistivity Tomography

- DC electrical current is injected through a pair of electrodes and the voltage is measured by the remaining ones.
- A line (array) of electrodes is used and a multiplexer sends current to different pairs of electrodes through a set sequence
- The distribution of electrical resistivity in the ground is obtained by tomographic reconstruction (inverse problem)



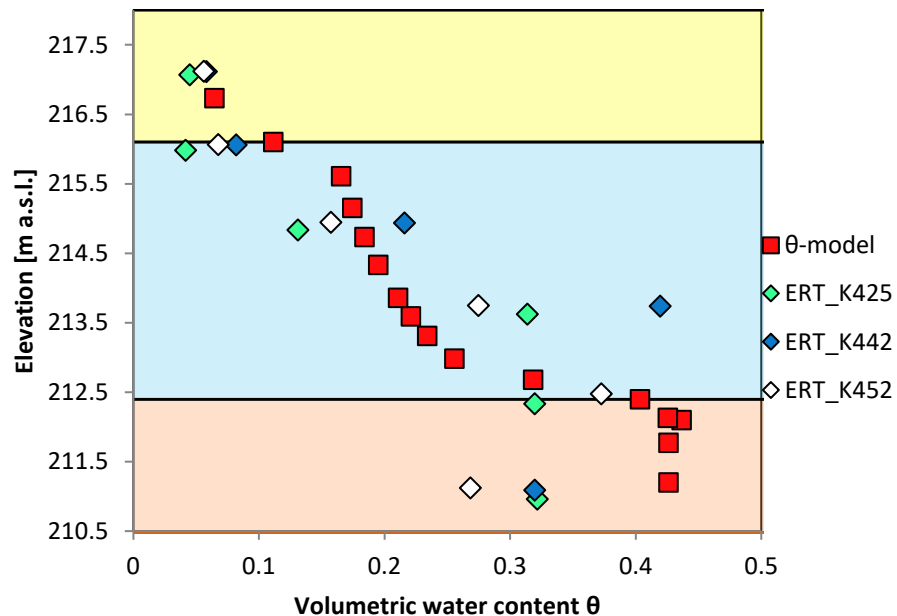
# Electrical resistivity mapping



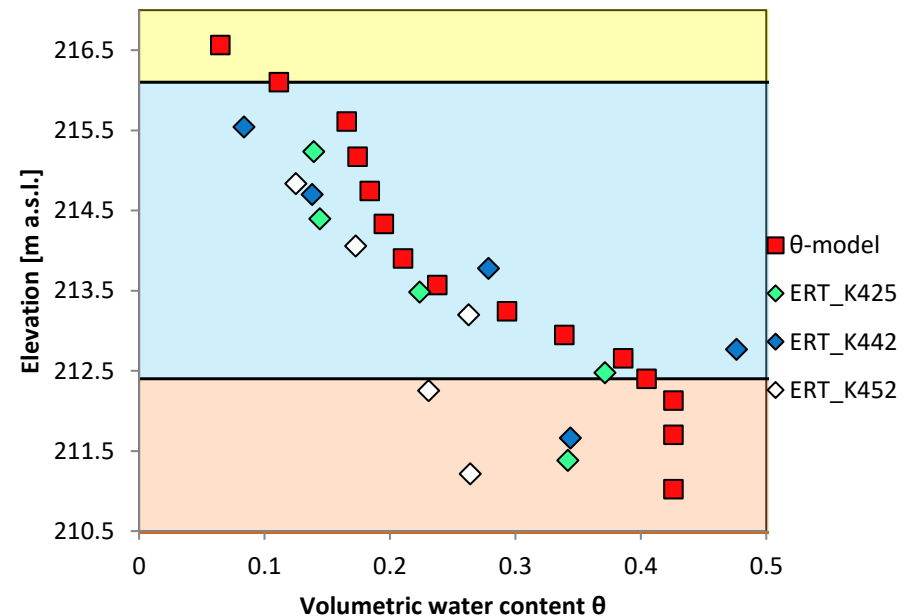
# Water content – simulation vs. ERT

## Water content profiles – November 2014

Crest\_l 2014



Crest\_r 2014



- Excellent match in the unsaturated embankment
- In deep foundation  $\theta = \theta_{\text{sat}}$